THE RANK 2 ROOTS PACKAGE
VERSION 1.2

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Contents

1. Introduction

This package concerns mathematical drawings arising in representation theory. The purpose of this package is to ease drawing of rank 2 root systems, with Weyl chambers, weight lattices, and parabolic subgroups, mostly imitating the drawings of Fulton and Harris [2]. We use definitions of root systems and weight lattices as in Carter [1] p. 540–609.

Load the `rank-2-roots` package

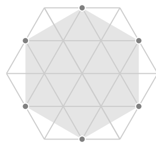
```
\documentclass{amsart}
\usepackage{rank-2-roots}
\begin{document}
The root system \(G_2\):
\begin{tikzpicture}[baseline=-.5]
\begin{rootSystem}{G}
\roots
\end{rootSystem}
\end{tikzpicture}
\end{document}
```

2. Root systems

Date: 3 September 2023.

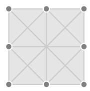
Table 1: The root systems

A_2



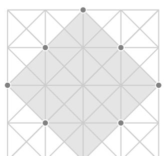
```
\begin {rootSystem}{A}
\roots
\roots
\end {rootSystem}
```

B_2



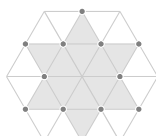
```
\begin {rootSystem}{B}
\roots
\roots
\end {rootSystem}
```

C_2



```
\begin {rootSystem}{C}
\roots
\roots
\end {rootSystem}
```

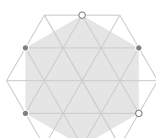
G_2



```
\begin {rootSystem}{G}
\roots
\roots
\end {rootSystem}
```

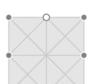
Table 2: The root systems with the simple roots marked

A_2



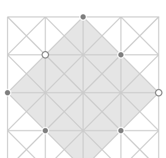
```
\begin {rootSystem}{A}
\roots
\simpleroots
\end {rootSystem}
```

B_2



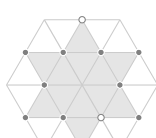
```
\begin {rootSystem}{B}
\roots
\simpleroots
\end {rootSystem}
```

C_2



```
\begin {rootSystem}{C}
\roots
\simpleroots
\end {rootSystem}
```

G_2



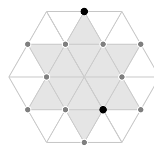
```
\begin {rootSystem}{G}
\roots
\simpleroots
\end {rootSystem}
```

continued . . .

Table 2: . . . continued

To change the style of the simple roots:

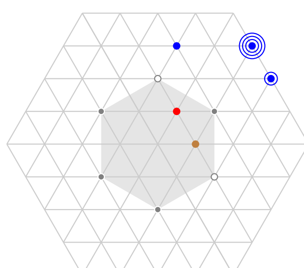
```
\pgfkeys{/root system/simple root/.style=black}
```

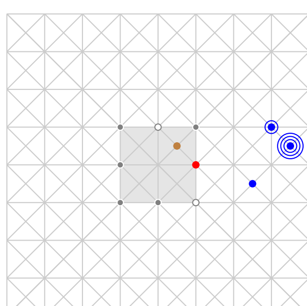


3. Weights

Type `\wt{x}{y}` to get a weight at position (x, y) (as measured in a basis of *fundamental weights*). Type `\wt[multiplicity=n]{x}{y}` to get multiplicity m. Add an option: `\wt[Z]{x}{y}` to get `Z` passed to TikZ.

Table 3: Some weights drawn with multiplicities

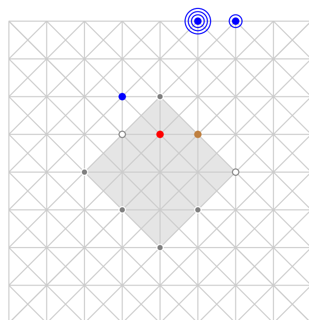
A_2		`\begin {rootSystem}{A}`
		`\roots`
		`\simpleroots`
		`\wt [brown]{1}{0}`
		`\wt [red]{0}{1}`
		`\wt [multiplicity=4,blue]{1}{3}`
		`\wt [blue,multiplicity=2]{2}{2}`
		`\wt [blue]{-1}{3}`
		`\end {rootSystem}`

B_2		`\begin {rootSystem}{B}`
		`\roots`
		`\simpleroots`
		`\wt [brown]{1}{0}`
		`\wt [red]{0}{1}`
		`\wt [multiplicity=4,blue]{1}{3}`
		`\wt [blue,multiplicity=2]{2}{2}`
		`\wt [blue]{-1}{3}`
		`\end {rootSystem}`

continued . . .

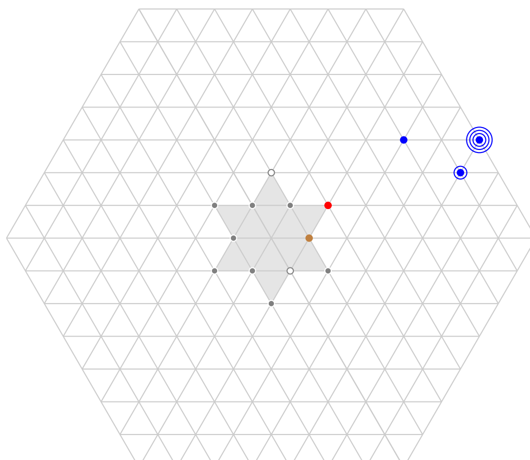
Table 3: . . . continued

C_2



```
\begin {rootSystem}{C}
\roots
\simpleroots
\wt [brown]{1}{0}
\wt [red]{0}{1}
\wt [multiplicity=4,blue]{1}{3}
\wt [blue,multiplicity=2]{2}{2}
\wt [blue]{-1}{3}
\end {rootSystem}
```

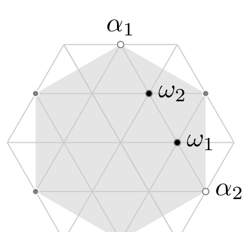
G_2



```
\begin {rootSystem}{G}
\roots
\simpleroots
\wt [brown]{1}{0}
\wt [red]{0}{1}
\wt [multiplicity=4,blue]{1}{3}
\wt [blue,multiplicity=2]{2}{2}
\wt [blue]{-1}{3}
\end {rootSystem}
```

Table 4: The fundamental weights and the simple roots

A_2

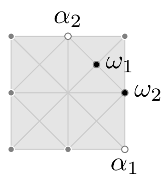



```
\begin{rootSystem}{A}
\roots
\simpleroots
\node [above] at \Root {1}{0} {\(\alpha_1\)};
\node [right] at \Root {0}{1} {\(\alpha_2\)};
\fundamentalweights
\node [right] at \weight {1}{0} {\(\omega_1\)};
\node [right] at \weight {0}{1} {\(\omega_2\)};
\end{rootSystem}
```

continued . . .

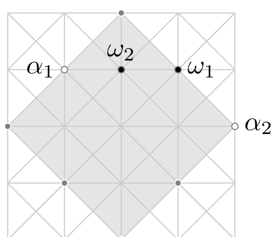
Table 4: . . . continued

B_2



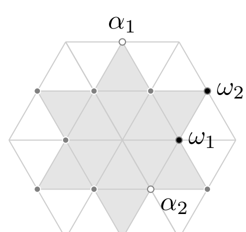

```
\begin{rootSystem}{B}
\roots
\simpleroots
\node [below] at \Root {1}{0} {\(\alpha_1\)};
\node [above] at \Root {0}{1} {\(\alpha_2\)};
\fundamentalweights
\node [right] at \weight {1}{0} {\(\omega_1\)};
\node [right] at \weight {0}{1} {\(\omega_2\)};
\end{rootSystem}{B}
```

C_2




```
\begin{rootSystem}{C}
\roots
\simpleroots
\node [left] at \Root {1}{0} {\(\alpha_1\)};
\node [right] at \Root {0}{1} {\(\alpha_2\)};
\fundamentalweights
\node [right] at \weight {1}{0} {\(\omega_1\)};
\node [above] at \weight {0}{1} {\(\omega_2\)};
\end{rootSystem}
```

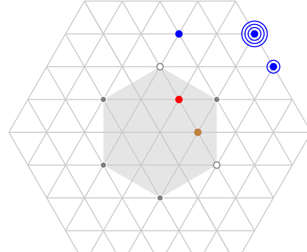
G_2




```
\begin{rootSystem}{G}
\roots
\simpleroots
\node [above] at \Root {1}{0} {\(\alpha_1\)};
\node [below right] at \Root {0}{1} {\(\alpha_2\)};
\fundamentalweights
\node [right] at \weight {1}{0} {\(\omega_1\)};
\node [right] at \weight {0}{1} {\(\omega_2\)};
\end{rootSystem}
```

Table 5: The root systems with all multiplicities of the adjoint representation, like Fulton and Harris

A_2

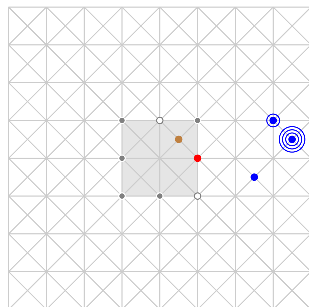


```
\begin {rootSystem}{A}
\roots
\wt [multiplicity=2,root]{0}{0}
\end {rootSystem}
```

continued . . .

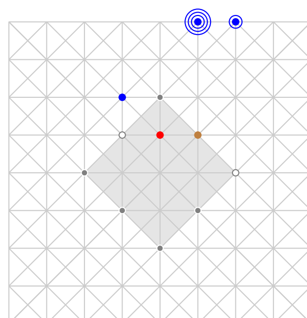
Table 5: ... continued

B_2



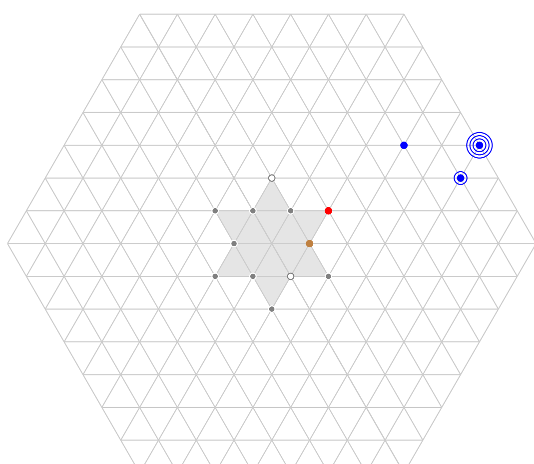
```
\begin {rootSystem}{B}
\roots
\wt [multiplicity=2,root]{0}{0}
\end {rootSystem}
```

C_2



```
\begin {rootSystem}{C}
\roots
\wt [multiplicity=2,root]{0}{0}
\end {rootSystem}
```

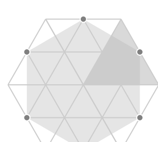
G_2



```
\begin {rootSystem}{G}
\roots
\wt [multiplicity=2,root]{0}{0}
\end {rootSystem}
```

Table 6: Weyl chambers

A_2

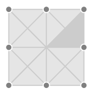


```
\begin {rootSystem}{A}
\roots
\WeylChamber
\end {rootSystem}
```

continued ...

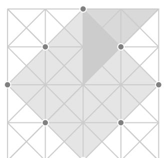
Table 6: …continued

B_2



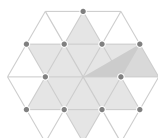
```
\begin {rootSystem}{B}
\roots
\WeylChamber
\end {rootSystem}
```

C_2



```
\begin {rootSystem}{C}
\roots
\WeylChamber
\end {rootSystem}
```

G_2

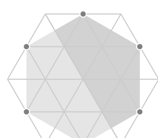


```
\begin {rootSystem}{G}
\roots
\WeylChamber
\end {rootSystem}
```

4. Parabolic subgroups

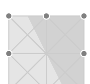
Table 7: The positive root hyperplane

A_2



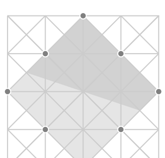
```
\begin {rootSystem}{A}
\roots
\positiveRootHyperplane
\end {rootSystem}
```

B_2



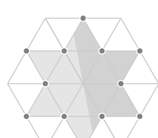
```
\begin {rootSystem}{B}
\roots
\positiveRootHyperplane
\end {rootSystem}
```

C_2



```
\begin {rootSystem}{C}
\roots
\positiveRootHyperplane
\end {rootSystem}
```

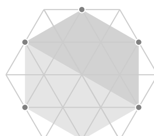
G_2



```
\begin {rootSystem}{G}
\roots
\positiveRootHyperplane
\end {rootSystem}
```

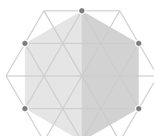
Table 8: Parabolic subgroups. Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not: $A_{5,37}$ means the parabolic subgroup of A_5 so that the binary digits of $37 = 2^5 + 2^2 + 2^0$ give us roots $0, 2, 5$ in Bourbaki ordering being crossed roots, i.e. noncompact roots, i.e. having the root vectors of that root but not of its negative inside the parabolic subgroup.

$A_{2,1}$



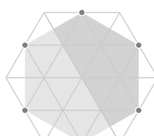
```
\begin {rootSystem}{A}
\roots
\parabolic {1}
\end {rootSystem}
```

$A_{2,2}$



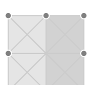
```
\begin {rootSystem}{A}
\roots
\parabolic {2}
\end {rootSystem}
```

$A_{2,3}$



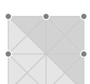
```
\begin {rootSystem}{A}
\roots
\parabolic {3}
\end {rootSystem}
```

$B_{2,1}$



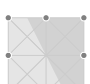
```
\begin {rootSystem}{B}
\roots
\parabolic {1}
\end {rootSystem}
```

$B_{2,2}$



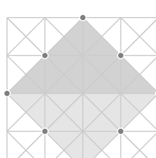
```
\begin {rootSystem}{B}
\roots
\parabolic {2}
\end {rootSystem}
```

$B_{2,3}$



```
\begin {rootSystem}{B}
\roots
\parabolic {3}
\end {rootSystem}
```

$C_{2,1}$

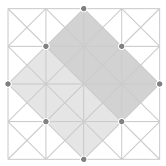


```
\begin {rootSystem}{C}
\roots
\parabolic {1}
\end {rootSystem}
```

continued . . .

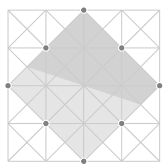
Table 8: . . . continued

$C_{2,2}$



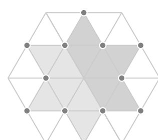
```
\begin {rootSystem}{C}
\roots
\parabolic {2}
\end {rootSystem}
```

$C_{2,3}$



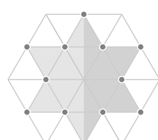
```
\begin {rootSystem}{C}
\roots
\parabolic {3}
\end {rootSystem}
```

$G_{2,1}$



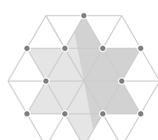
```
\begin {rootSystem}{G}
\roots
\parabolic {1}
\end {rootSystem}
```

$G_{2,2}$



```
\begin {rootSystem}{G}
\roots
\parabolic {2}
\end {rootSystem}
```

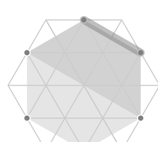
$G_{2,3}$



```
\begin {rootSystem}{G}
\roots
\parabolic {3}
\end {rootSystem}
```

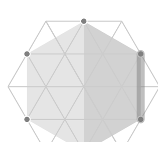
Table 9: Parabolic subgroups with grading of the positive roots

$A_{2,1}$



```
\begin {rootSystem}{A}
\roots
\parabolic {1}
\parabolicgrading
\end {rootSystem}
```

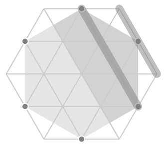
$A_{2,2}$

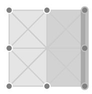


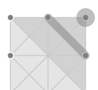
```
\begin {rootSystem}{A}
\roots
\parabolic {2}
\parabolicgrading
\end {rootSystem}
```

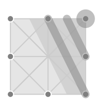
continued . . .

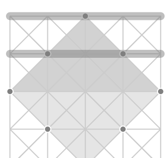
Table 9: . . . continued

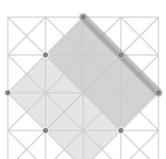
$A_{2,3}$		`\begin {rootSystem}{A}`
		`\roots`
		`\parabolic {3}`
		`\parabolicgrading`
		`\end {rootSystem}`

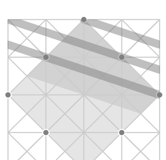
$B_{2,1}$		`\begin {rootSystem}{B}`
		`\roots`
		`\parabolic {1}`
		`\parabolicgrading`
		`\end {rootSystem}`

$B_{2,2}$		`\begin {rootSystem}{B}`
		`\roots`
		`\parabolic {2}`
		`\parabolicgrading`
		`\end {rootSystem}`

$B_{2,3}$		`\begin {rootSystem}{B}`
		`\roots`
		`\parabolic {3}`
		`\parabolicgrading`
		`\end {rootSystem}`

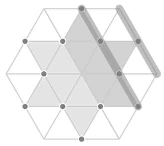
$C_{2,1}$		`\begin {rootSystem}{C}`
		`\roots`
		`\parabolic {1}`
		`\parabolicgrading`
		`\end {rootSystem}`

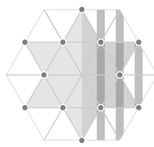
$C_{2,2}$		`\begin {rootSystem}{C}`
		`\roots`
		`\parabolic {2}`
		`\parabolicgrading`
		`\end {rootSystem}`

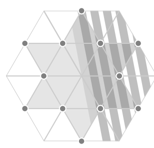
$C_{2,3}$		`\begin {rootSystem}{C}`
		`\roots`
		`\parabolic {3}`
		`\parabolicgrading`
		`\end {rootSystem}`

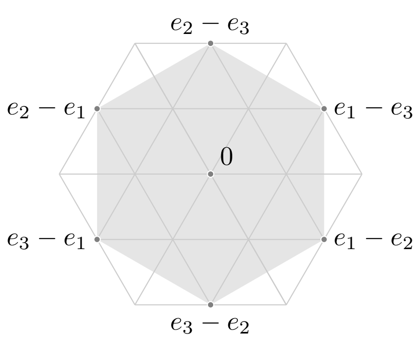
continued . . .

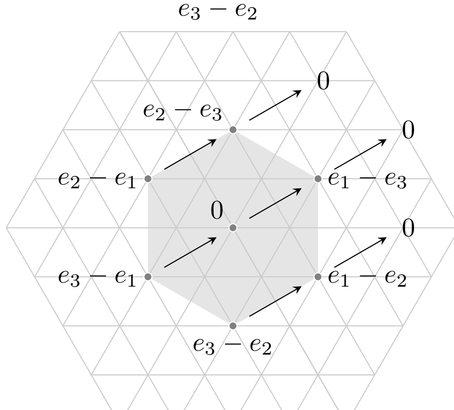
Table 9: . . . continued

$G_{2,1}$		`\begin {rootSystem}{G}`
		`\roots`
		`\parabolic {1}`
		`\parabolicgrading`
		`\end {rootSystem}`

$G_{2,2}$		`\begin {rootSystem}{G}`
		`\roots`
		`\parabolic {2}`
		`\parabolicgrading`
		`\end {rootSystem}`

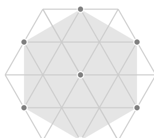
$G_{2,3}$		`\begin {rootSystem}{G}`
		`\roots`
		`\parabolic {3}`
		`\parabolicgrading`
		`\end {rootSystem}`





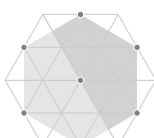
Drawing the A_2 root system and a weight at the origin. The option `root` indicates that this weight is to be coloured like a root.

```
\begin{tikzpicture}
\begin{rootSystem}{A}
\roots
\wt[root]{0}{0}
\end{rootSystem}
\end{tikzpicture}
```



Drawing the A_2 root system and a weight at the origin and the positive root hyperplane

```
\begin{tikzpicture}
\begin{rootSystem}{A}
\roots
\wt[root]{0}{0}
\positiveRootHyperplane
\end{rootSystem}
\end{tikzpicture}
```



5. Coordinate systems

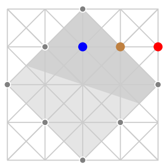
The package provides three coordinate systems: hex, square and weight. Above we have seen the weight coordinates: a basis of fundamental weights. We can also use weight coordinates like

```
\draw \weight{0}{1} -- \weight{1}{0};
```

Drawing weights as linear combinations of fundamental weights

```
\begin{tikzpicture}
\begin{rootSystem}{C}
\roots
\positiveRootHyperplane
\fill[thick,brown] \weight{1}{0} circle (1.7pt);
\fill[thick,blue] \weight{0}{1} circle (1.7pt);
```

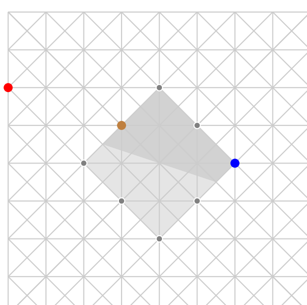
```
\fill[thick,red] \weight{2}{-1} circle (1.7pt);
\end{rootSystem}
\end{tikzpicture}
```



We can also specify roots in linear combinations of the simple roots:

Drawing roots as linear combinations of simple roots

```
\begin{tikzpicture}
\begin{rootSystem}{C}
\roots
\positiveRootHyperplane
\fill[thick,brown] \Root{1}{0} circle (1.7pt);
\fill[thick,blue] \Root{0}{1} circle (1.7pt);
\fill[thick,red] \Root{2}{-1} circle (1.7pt);
\end{rootSystem}
\end{tikzpicture}
```

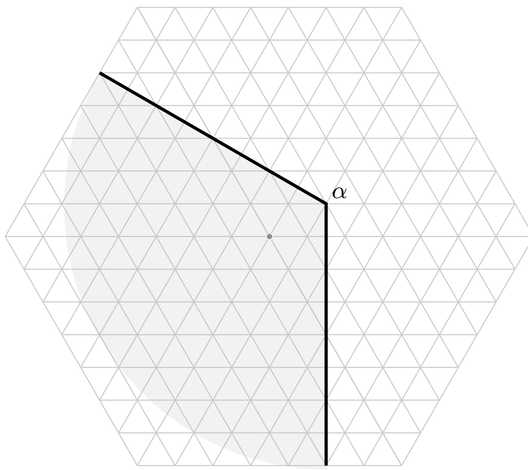


The square system, used like `\draw (square cs:x=1,y=2) circle (2pt);`, is simply the standard Cartesian coordinate system measured so that the minimum distance between weights is one unit. The hex coordinate system has basis precisely the fundamental weights of the A_2 lattice. We can use the hex system in drawing on the A_2 or G_2 weight lattices, as below, as they are the same lattices.

Automatic sizing of the weight lattice (the default) . . .

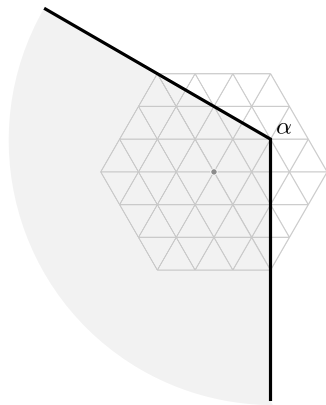
```
\begin{tikzpicture}
\begin{rootSystem}{A}
\wt{0}{0}
```

```
\fill[gray!50,opacity=.2]  (hex cs:x=5,y=-7) -- (hex cs:x=1,y=1) --
    (hex cs:x=-7,y=5) arc (150:270:{7*\weightLength});
\draw[black,very thick] (hex cs:x=5,y=-7) -- (hex cs:x=1,y=1) --
    (hex cs:x=-7,y=5);
\node[above right=-2pt] at (hex cs:x=1,y=1) {\small\(\alpha\)};
\end{rootSystem}
\end{tikzpicture}
```



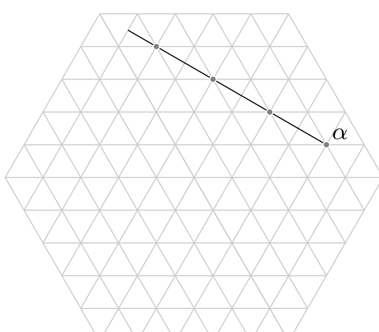
. . . and here with manual sizing, setting the weight lattice to include 3 steps to the right of the origin

```
\begin{tikzpicture}
\AutoSizeWeightLatticefalse
\begin{rootSystem}{A}
\wt{0}{0}
\weightLattice{3}
\fill[gray!50,opacity=.2]  (hex cs:x=5,y=-7) -- (hex cs:x=1,y=1) --
    (hex cs:x=-7,y=5) arc (150:270:{7*\weightLength});
\draw[black,very thick] (hex cs:x=5,y=-7) -- (hex cs:x=1,y=1) --
    (hex cs:x=-7,y=5);
\node[above right=-2pt] at (hex cs:x=1,y=1) {\small\(\alpha\)};
\end{rootSystem}
\end{tikzpicture}
```



Fulton and Harris p. 170

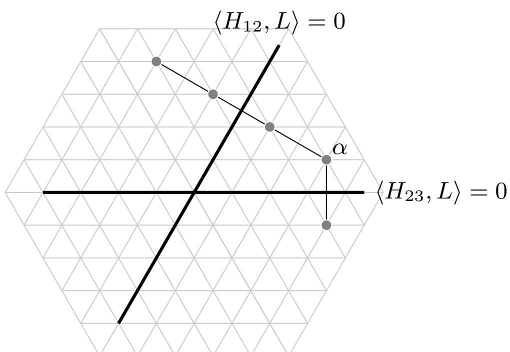
```
\begin{tikzpicture}
\begin{rootSystem}{A}
\draw \weight{3}{1} -- \weight{-4}{4.5};
\foreach \i in {1,...,4}{\wt{5-2*\i}{\i}}
\node[above right=-2pt] at (hex cs:x=3,y=1){\small\(\alpha\)};
\end{rootSystem}
\end{tikzpicture}
```



Automatic sizing of the weight lattice (the default) . . .

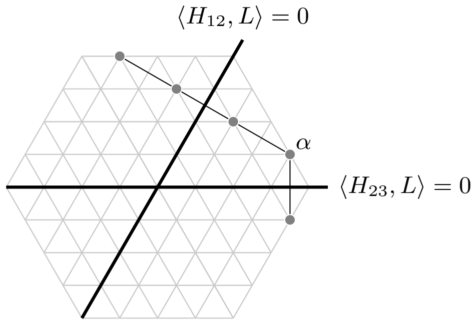
```
\begin{tikzpicture}
\begin{rootSystem}{A}
\setlength{\weightRadius}{2pt}
\draw \weight{3}{1} -- \weight{-3}{4};
\draw \weight{3}{1} -- \weight{4}{-1};
\wt{4}{-1}
\foreach \i in {1,...,4}{\wt{5-2*\i}{\i}}
\node[above right=-2pt] at (hex cs:x=3,y=1){\small\(\alpha\)};
```

```
\draw[very thick] \weight{0}{-4} -- \weight{0}{4.5}
    node[above]{\small\(\left<H_{12},L\right>=0\)};
\draw[very thick] \weight{-4}{0} -- \weight{4.5}{0}
    node[right]{\small\(\left<H_{23},L\right>=0\)};
\end{rootSystem}
\end{tikzpicture}
```

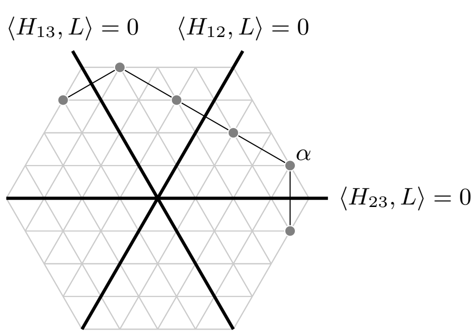


... and manual sizing

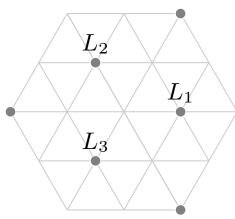
```
\begin{tikzpicture}
\AutoSizeWeightLatticefalse
\begin{rootSystem}{A}
\setlength{\weightRadius}{2pt}
\weightLattice{4}
\draw \weight{3}{1} -- \weight{-3}{4};
\draw \weight{3}{1} -- \weight{4}{-1};
\wt{4}{-1}
\foreach \i in {1,...,4}{\wt{5-2*\i}{\i}}
\node[above right=-2pt] at (hex cs:x=3,y=1){\small\(\alpha\)};
\draw[very thick] \weight{0}{-4} -- \weight{0}{4.5}
    node[above]{\small\(\left<H_{12},L\right>=0\)};
\draw[very thick] \weight{-4}{0} -- \weight{4.5}{0}
    node[right]{\small\(\left<H_{23},L\right>=0\)};
\end{rootSystem}
\end{tikzpicture}
```




```
\begin{tikzpicture}
\AutoSizeWeightLatticefalse
\begin{rootSystem}{A}
\setlength{\weightRadius}{2pt}
\weightLattice{4}
\draw \weight{3}{1} -- \weight{-3}{4};
\draw \weight{3}{1} -- \weight{4}{-1};
\draw \weight{-3}{4} -- \weight{-4}{3};
\wt{4}{-1}
\wt{-4}{3}
\foreach \i in {1,...,4}{\wt{5-2*\i}{\i}}
\node[above right=-2pt] at (hex cs:x=3,y=1){\small\(\alpha\)};
\draw[very thick] \weight{0}{-4} -- \weight{0}{4.5}
    node[above]{\small\(\left<H_{12},L\right>=0\)};
\draw[very thick] \weight{-4}{0} -- \weight{4.5}{0}
    node[right]{\small\(\left<H_{23},L\right>=0\)};
\draw[very thick] \weight{4}{-4} -- \weight{-4.5}{4.5}
    node[above]{\small\(\left<H_{13},L\right>=0\)};
\end{rootSystem}
\end{tikzpicture}
```

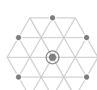


```
\setlength{\weightRadius}{2pt}
\setlength\weightLength{.75cm}
\begin{tikzpicture}
\begin{rootSystem}{A}
\foreach \x/\y in {1/0, -1/1, 0/-1, -2/0, 0/2, 2/-2}{\wt{\x}{\y}}
\node[above] at \weight{1}{0} {\small\(L_1\)};
\node[above] at \weight{-1}{1} {\small\(L_2\)};
\node[above] at \weight{0}{-1} {\small\(L_3\)};
\end{rootSystem}
\end{tikzpicture}
```



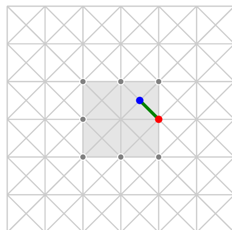
Changing the weight length rescales

```
\begin{tikzpicture}
\pgfkeys{/root system/weight length=0.3cm}
\begin{rootSystem}{A}
\wt[multiplicity=2,draw=gray]{0}{0}
\foreach \x/\y in {1/1, 2/-1, 1/-2, -1/-1, -2/1, -1/2}{\wt{\x}{\y}}
\end{rootSystem}
\end{tikzpicture}
```



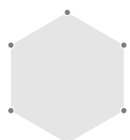
We use a basis of fundamental weights, as given in Carter's book [1] p. 540–609

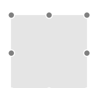
```
\begin{tikzpicture}
\begin{rootSystem}{B}
\roots
\draw[green!50!black,very thick] \weight{0}{1} -- \weight{1}{0};
\weightLattice{3}
\wt[blue]{1}{0}
\wt[red]{0}{1}
\end{rootSystem}
\end{tikzpicture}
```

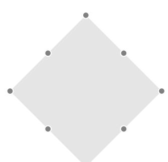


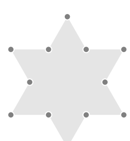
Without automatic stretching of the weight lattice to fit the picture, you won't see the weight lattice at all unless you ask for it.

Table 10: The root systems

A_2		`\begin {rootSystem}{A}`
		`\roots`
		`\end {rootSystem}`

B_2		`\begin {rootSystem}{B}`
		`\roots`
		`\end {rootSystem}`

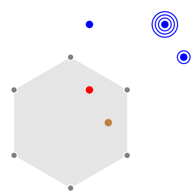
C_2		`\begin {rootSystem}{C}`
		`\roots`
		`\end {rootSystem}`

G_2		`\begin {rootSystem}{G}`
		`\roots`
		`\end {rootSystem}`

Type `\wt{x}{y}` to get a weight at position (x, y) (as measured in a basis of *fundamental weights*). Add an option: `\wt[Z]{x}{y}` to get `Z` passed to TikZ, or with option `multiplicity=n` to get multiplicity n.

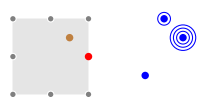
Table 11: Some weights drawn with multiplicities

A_2



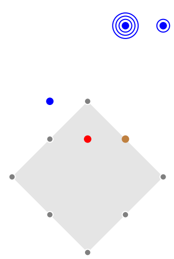
```
\begin {rootSystem}{A}
\roots
\wt [brown]{1}{0}
\wt [red]{0}{1}
\wt [blue,multiplicity=4]{1}{3}
\wt [blue,multiplicity=2]{2}{2}
\wt [blue]{-1}{3}
\end {rootSystem}
```

B_2



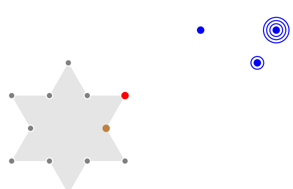
```
\begin {rootSystem}{B}
\roots
\wt [brown]{1}{0}
\wt [red]{0}{1}
\wt [blue,multiplicity=4]{1}{3}
\wt [blue,multiplicity=2]{2}{2}
\wt [blue]{-1}{3}
\end {rootSystem}
```

C_2



```
\begin {rootSystem}{C}
\roots
\wt [brown]{1}{0}
\wt [red]{0}{1}
\wt [blue,multiplicity=4]{1}{3}
\wt [blue,multiplicity=2]{2}{2}
\wt [blue]{-1}{3}
\end {rootSystem}
```

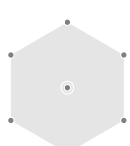
G_2



```
\begin {rootSystem}{G}
\roots
\wt [brown]{1}{0}
\wt [red]{0}{1}
\wt [blue,multiplicity=4]{1}{3}
\wt [blue,multiplicity=2]{2}{2}
\wt [blue]{-1}{3}
\end {rootSystem}
```

Table 12: The root systems with all multiplicities of the adjoint representation, like Fulton and Harris

A_2

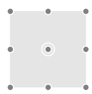


```
\begin {rootSystem}{A}
\roots
\wt [multiplicity=2]{0}{0}
\end {rootSystem}
```

continued . . .

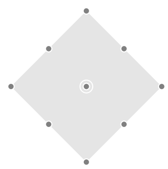
Table 12: . . . continued

B_2



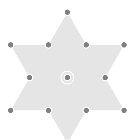
```
\begin {rootSystem}{B}
\roots
\wt [multiplicity=2]{0}{0}
\end {rootSystem}
```

C_2



```
\begin {rootSystem}{C}
\roots
\wt [multiplicity=2]{0}{0}
\end {rootSystem}
```

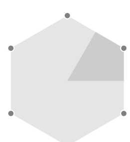
G_2



```
\begin {rootSystem}{G}
\roots
\wt [multiplicity=2]{0}{0}
\end {rootSystem}
```

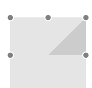
Table 13: Weyl chambers

A_2



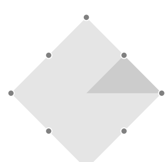
```
\begin {rootSystem}{A}
\roots
\WeylChamber
\end {rootSystem}
```

B_2



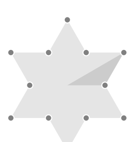
```
\begin {rootSystem}{B}
\roots
\WeylChamber
\end {rootSystem}
```

C_2



```
\begin {rootSystem}{C}
\roots
\WeylChamber
\end {rootSystem}
```

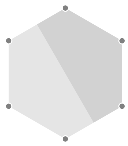
G_2



```
\begin {rootSystem}{G}
\roots
\WeylChamber
\end {rootSystem}
```

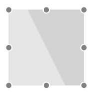
Table 14: The positive root hyperplane

A_2



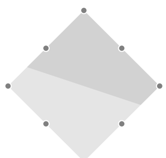
```
\begin {rootSystem}{A}
\roots
\positiveRootHyperplane
\end {rootSystem}
```

B_2



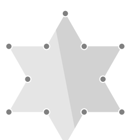
```
\begin {rootSystem}{B}
\roots
\positiveRootHyperplane
\end {rootSystem}
```

C_2



```
\begin {rootSystem}{C}
\roots
\positiveRootHyperplane
\end {rootSystem}
```

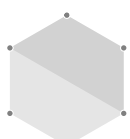
G_2



```
\begin {rootSystem}{G}
\roots
\positiveRootHyperplane
\end {rootSystem}
```

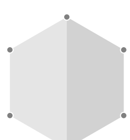
Table 15: Parabolic subgroups

$A_{2,1}$



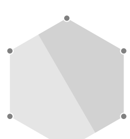
```
\begin {rootSystem}{A}
\roots
\parabolic {1}
\end {rootSystem}
```

$A_{2,2}$



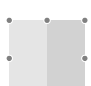
```
\begin {rootSystem}{A}
\roots
\parabolic {2}
\end {rootSystem}
```

$A_{2,3}$



```
\begin {rootSystem}{A}
\roots
\parabolic {3}
\end {rootSystem}
```

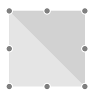
$B_{2,1}$

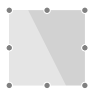


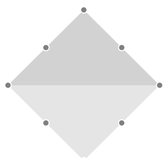
```
\begin {rootSystem}{B}
\roots
\parabolic {1}
\end {rootSystem}
```

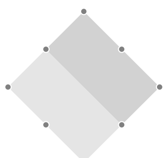
continued . . .

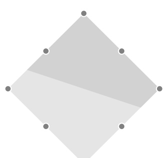
Table 15: ...continued

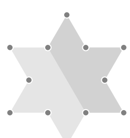
$B_{2,2}$		`\begin {rootSystem}{B}`
		`\roots`
		`\parabolic {2}`
		`\end {rootSystem}`

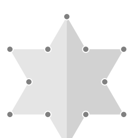
$B_{2,3}$		`\begin {rootSystem}{B}`
		`\roots`
		`\parabolic {3}`
		`\end {rootSystem}`

$C_{2,1}$		`\begin {rootSystem}{C}`
		`\roots`
		`\parabolic {1}`
		`\end {rootSystem}`

$C_{2,2}$		`\begin {rootSystem}{C}`
		`\roots`
		`\parabolic {2}`
		`\end {rootSystem}`

$C_{2,3}$		`\begin {rootSystem}{C}`
		`\roots`
		`\parabolic {3}`
		`\end {rootSystem}`

$G_{2,1}$		`\begin {rootSystem}{G}`
		`\roots`
		`\parabolic {1}`
		`\end {rootSystem}`

$G_{2,2}$		`\begin {rootSystem}{G}`
		`\roots`
		`\parabolic {2}`
		`\end {rootSystem}`

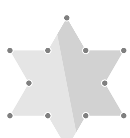
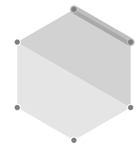
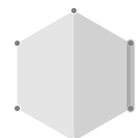
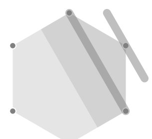
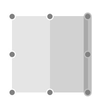
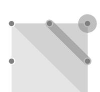
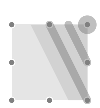
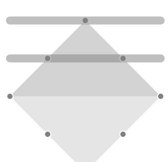
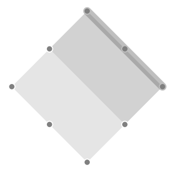
$G_{2,3}$		`\begin {rootSystem}{G}`
		`\roots`
		`\parabolic {3}`
		`\end {rootSystem}`

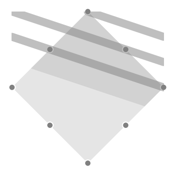
Table 16: Parabolic subgroups with grading of the positive roots

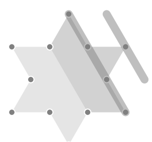
$A_{2,1}$		\begin {rootSystem}{A} \roots \parabolic {1} \parabolicgrading \end {rootSystem}
$A_{2,2}$		\begin {rootSystem}{A} \roots \parabolic {2} \parabolicgrading \end {rootSystem}
$A_{2,3}$		\begin {rootSystem}{A} \roots \parabolic {3} \parabolicgrading \end {rootSystem}
$B_{2,1}$		\begin {rootSystem}{B} \roots \parabolic {1} \parabolicgrading \end {rootSystem}
$B_{2,2}$		\begin {rootSystem}{B} \roots \parabolic {2} \parabolicgrading \end {rootSystem}
$B_{2,3}$		\begin {rootSystem}{B} \roots \parabolic {3} \parabolicgrading \end {rootSystem}
$C_{2,1}$		\begin {rootSystem}{C} \roots \parabolic {1} \parabolicgrading \end {rootSystem}

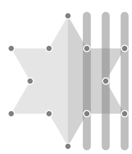
continued . . .

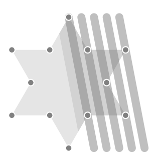
Table 16: . . . continued

$C_{2,2}$		`\begin {rootSystem}{C}`
		`\roots`
		`\parabolic {2}`
		`\parabolicgrading`
		`\end {rootSystem}`

$C_{2,3}$		`\begin {rootSystem}{C}`
		`\roots`
		`\parabolic {3}`
		`\parabolicgrading`
		`\end {rootSystem}`

$G_{2,1}$		`\begin {rootSystem}{G}`
		`\roots`
		`\parabolic {1}`
		`\parabolicgrading`
		`\end {rootSystem}`

$G_{2,2}$		`\begin {rootSystem}{G}`
		`\roots`
		`\parabolic {2}`
		`\parabolicgrading`
		`\end {rootSystem}`

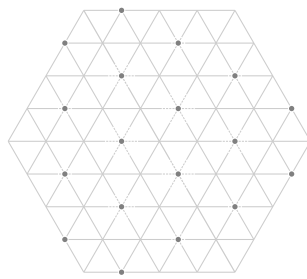
$G_{2,3}$		`\begin {rootSystem}{G}`
		`\roots`
		`\parabolic {3}`
		`\parabolicgrading`
		`\end {rootSystem}`

6. Examples of weights of various representations

Henceforth assume `\AutoSizeWeightLatticetrue` (the default).

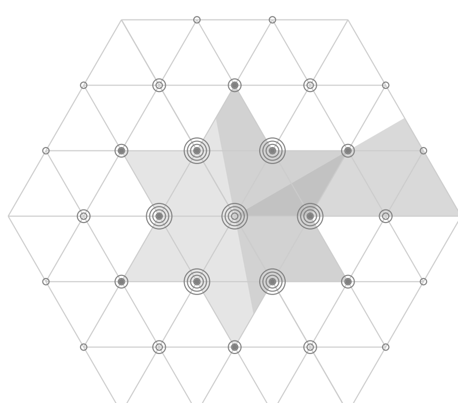
Fulton and Harris, p. 186

```
\begin{tikzpicture}
\begin{rootSystem}{A}
\foreach \x/\y/\m in
{0/ 1/5,  -1/0/5,  1/-1/5,  2/ 0/4, -2/ 2/4,  0/-2/4,
 1/ 2/2,  -1/3/2,  3/-2/2,  2/-3/2, -2/-1/2, -3/ 1/2,
 4/-1/1,   3/1/1, -3/ 4/1, -4/ 3/1, -1/-3/1,  1/-4/1}
{\wt[multiplicity=\m]{\x}{\y}}
\end{rootSystem}
\end{tikzpicture}
```



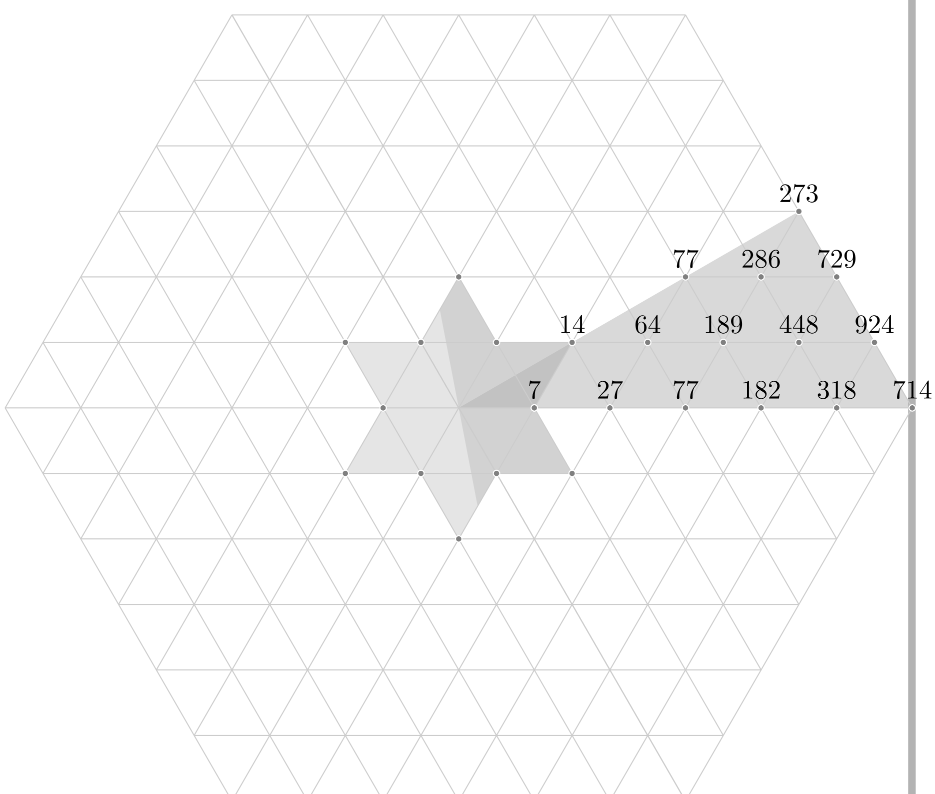
A representation of G_2

```
\begin{tikzpicture}
\begin{rootSystem}[weight
    length=1cm,weight/.style={draw=gray,fill=none}]{G}
\roots
\foreach \m/\x/\y in {
1/1/1, 1/4/-1, 1/-1/2, 2/2/0, 1/5/-2,
2/0/1, 2/3/-1, 2/-2/2, 4/1/0, 1/-4/3,
2/4/-2, 4/-1/1, 4/2/-1,  2/-3/2, 1/5/-3,
4/0/0, 1/-5/3, 2/3/-2, 4/-2/1, 4/1/-1,
2/-4/2, 1/4/-3, 4/-1/0, 2/2/-2, 2/-3/1,
2/0/-1, 1/-5/2, 2/-2/0, 1/1/-2, 1/-4/1,
1/-1/-1}{\wt[multiplicity=\m]{\x}{\y}}
\positiveRootHyperplane
\WeylChamber
\end{rootSystem}
\end{tikzpicture}
```



Dimensions of representations of G_2, parameterized by highest weight

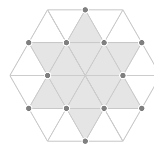
```
\begin{tikzpicture}
\begin{rootSystem}[weight length=1cm]{G}
\roots
\foreach \x/\y/\d in {
0/1/14, 0/2/77, 0/3/273, 1/0/7, 1/1/64,
1/2/286, 2/0/27, 2/1/189, 2/2/729, 3/0/77,
4/0/182, 5/0/318, 6/0/714, 3/1/448, 4/1/924}
{\wt{\x}{\y}\node[black,above] at \weight{\x}{\y} {\(\d\)};}
\positiveRootHyperplane
\WeylChamber
\end{rootSystem}
\end{tikzpicture}
```



7. More options

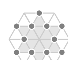
Options can be set using global TikZ keys

```
\pgfkeys{/root system/simple root/.style=black}
```



or, in each root system, using

```
\[
\begin{tikzpicture}
\begin{rootSystem}[weight length=.2cm]{G}
\roots
\end{rootSystem}
\end{tikzpicture}
\]
```



`weight radius`: length,
default = `1.2pt`
  Radius of dots used when marking specified weights.

`weight length`: length,
default = `.5cm`
  Minimum distance between distinct weights.

`grading dot radius`: length,
default = `2pt`
  Size of dot around a root using to indicate a grading of a parabolic subalgebra which only contains one root.

`weight lattice`: TikZ style data,
default = `gray!40`
  Style for drawing weight lattice lines.

`root`: TikZ style data,
default = `gray`
  Style for drawing roots.

`simple root`: TikZ style data,
default = `fill=white,draw=gray`
  Style for drawing simple roots.

`weight`: TikZ style data,
default = `fill=gray,draw=white`
  Style for drawing weights.

`fundamental weight`: TikZ style data,
default = `fill=black,draw=gray`
  Style for drawing fundamental weights.

`root polygon`: TikZ style data,
default = `gray!40,opacity=.5`

continued . . .

Table 17: ... continued

Style for drawing a polygon which indicates the locations of the roots.

`hyperplane`: TikZ style data,
default = `gray!50,fill opacity=.5`

Style for drawing a hyperplane in a root system which contains either the positive roots, or (more generally) the positive height roots of a parabolic subgroup.

`Weyl chamber`: TikZ style data,
default = `gray!60,fill opacity=.5`

Style for drawing a wedge indicating the Weyl chamber of a root system.

`grading`: TikZ style data,
default = `line width=3pt,gray,opacity=0.5,line cap=round`

Style for drawing a thick line over top of some roots to indicate that they lie in the same grading associated to a parabolic subgroup.

References

1. R. W. Carter, *Lie algebras of finite and affine type*, Cambridge Studies in Advanced Mathematics, vol. 96, Cambridge University Press, Cambridge, 2005. MR 2188930
2. William Fulton and Joe Harris, *Representation theory*, Graduate Texts in Mathematics, vol. 129, Springer-Verlag, New York, 1991, A first course, Readings in Mathematics. MR 1153249

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