Inside ElmerSolver

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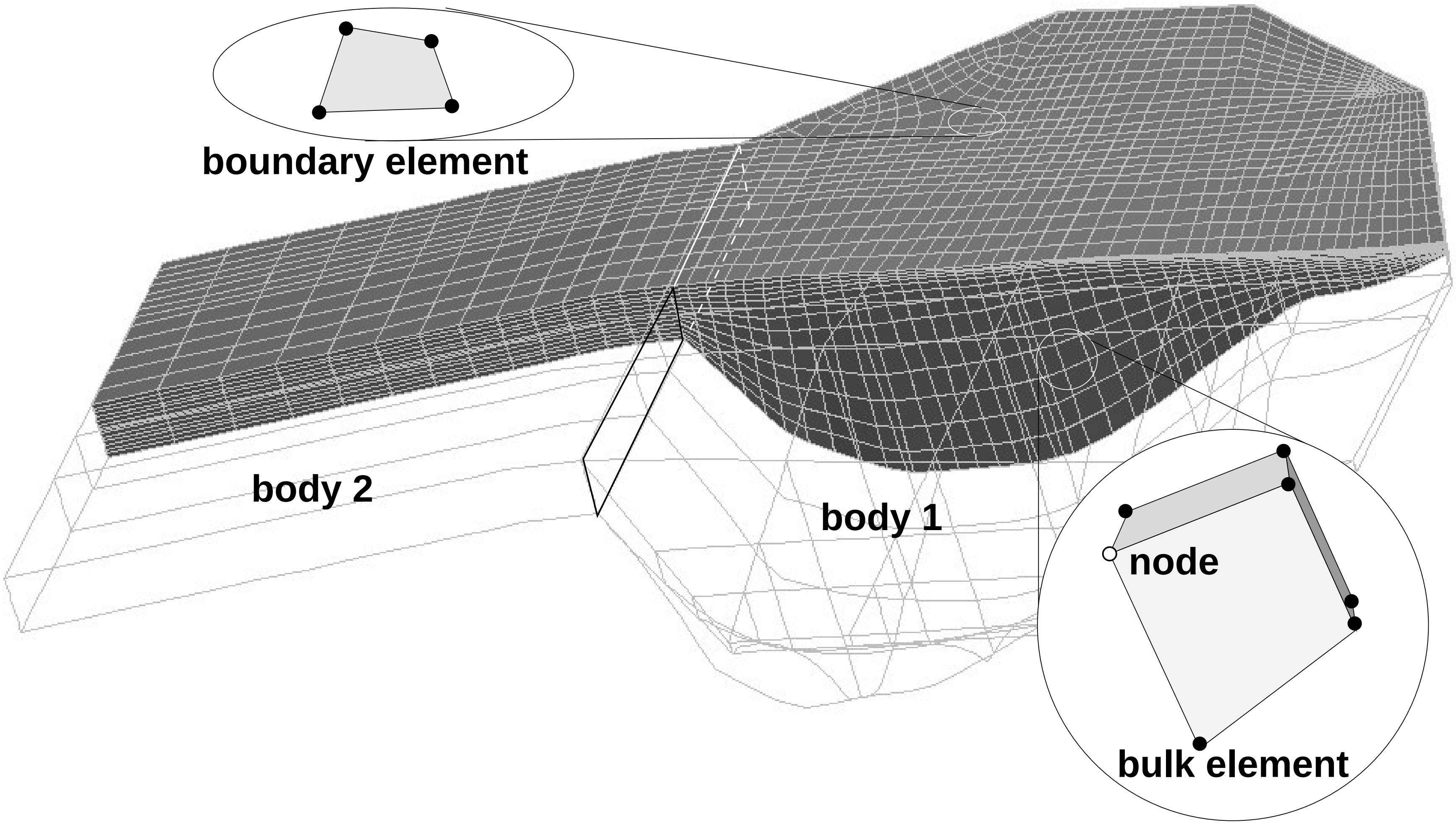
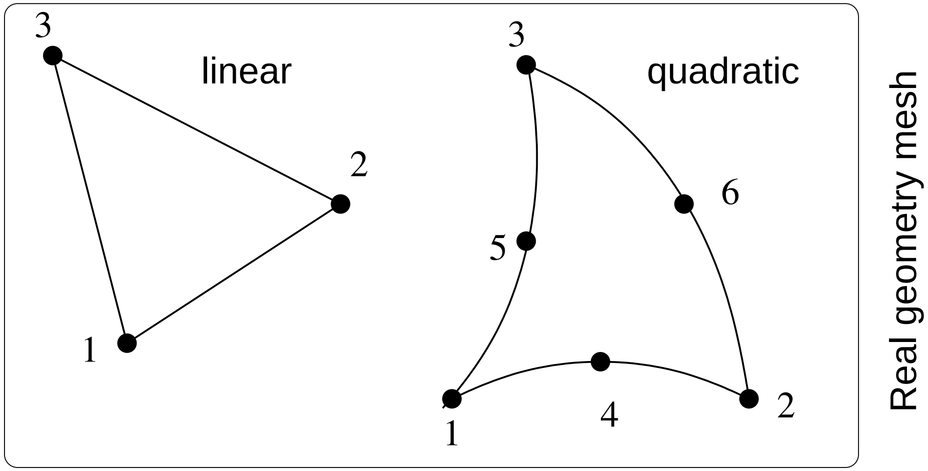
Espoo, Finland

Contents

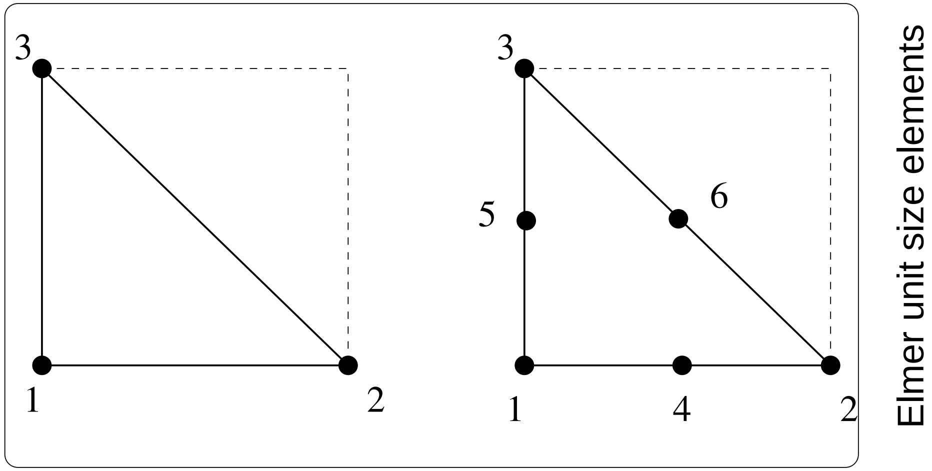
On Bodies and Boundaries



Finite Elements



+ Coordinate−system metric



Finite Elements contd.

General advection diffusion equation:

$$\underbrace{\Psi_\beta \, a(\Delta t) \int\limits_\Omega \phi_\beta \phi_\alpha d\Omega}_{\mathbf{M}} + \underbrace{\Psi_\beta \int\limits_\Omega \left[\mathbf{u} \cdot \nabla \phi_\beta \phi_\alpha + \kappa \nabla \phi_\beta \cdot \nabla \phi_\alpha\right] d\Omega}_{\mathbf{S}} =$$

$$\underbrace{\oint\limits_{\partial\Omega} (\kappa \nabla \Psi \phi_\alpha) \cdot \mathbf{n} d\Omega}_{\text{nat. BC}} + \underbrace{\int\limits_\Omega \sigma \phi_\alpha d\Omega}_{\mathbf{f}}$$

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Finite Elements contd.

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$$(\mathbf{M} + \mathbf{S}) \cdot \mathbf{\Psi} = \mathbf{f}$$

$\mathbf{M}\ldots$ Mass matrix, $\mathbf{S}\ldots$ Stiffness matrix, $\mathbf{f}\ldots$ force vector

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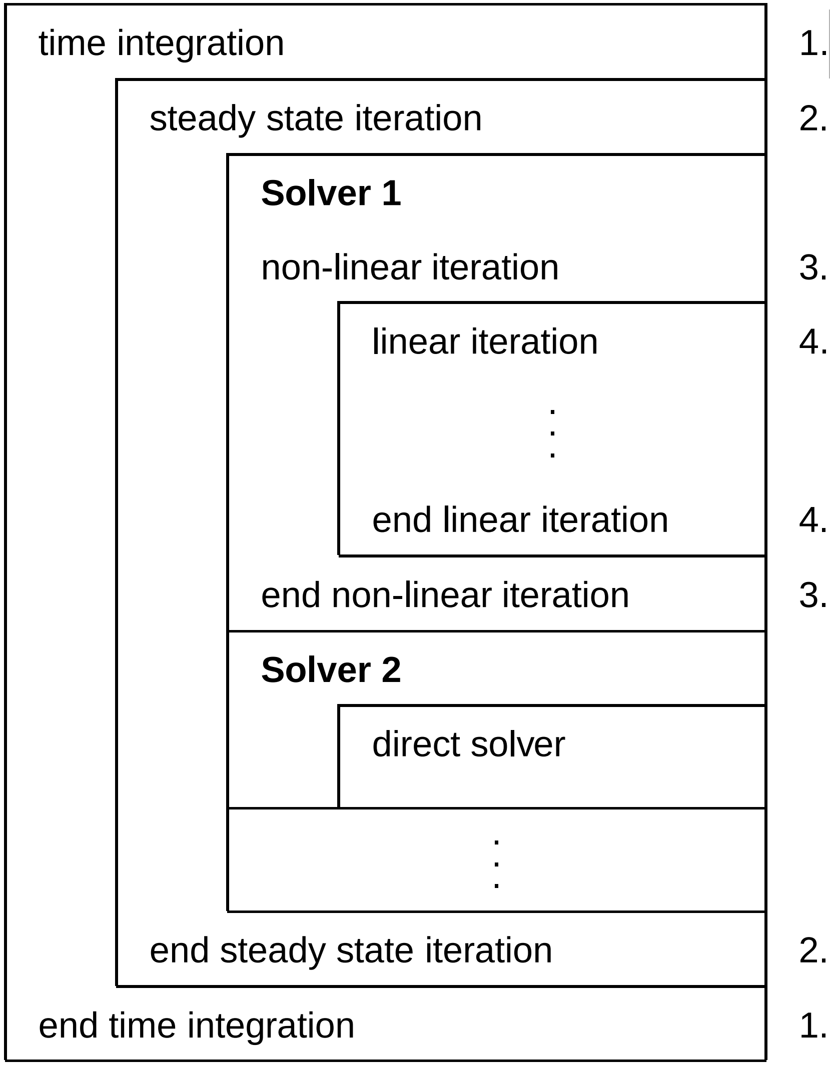
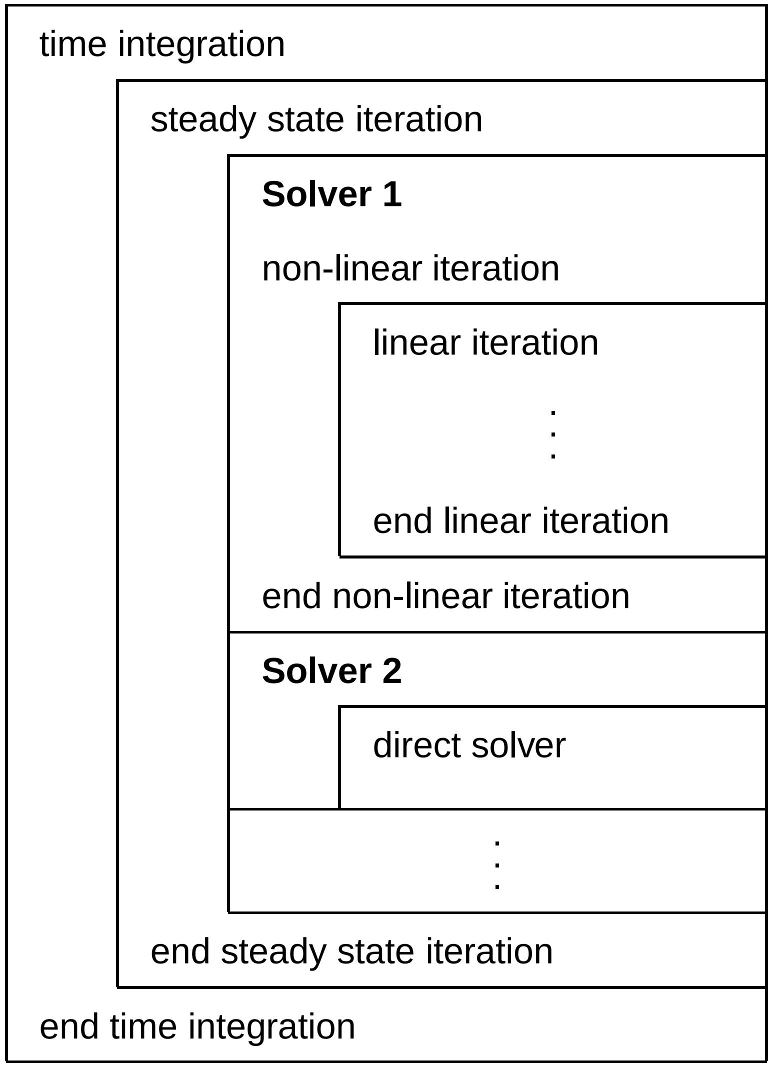
Solution Levels



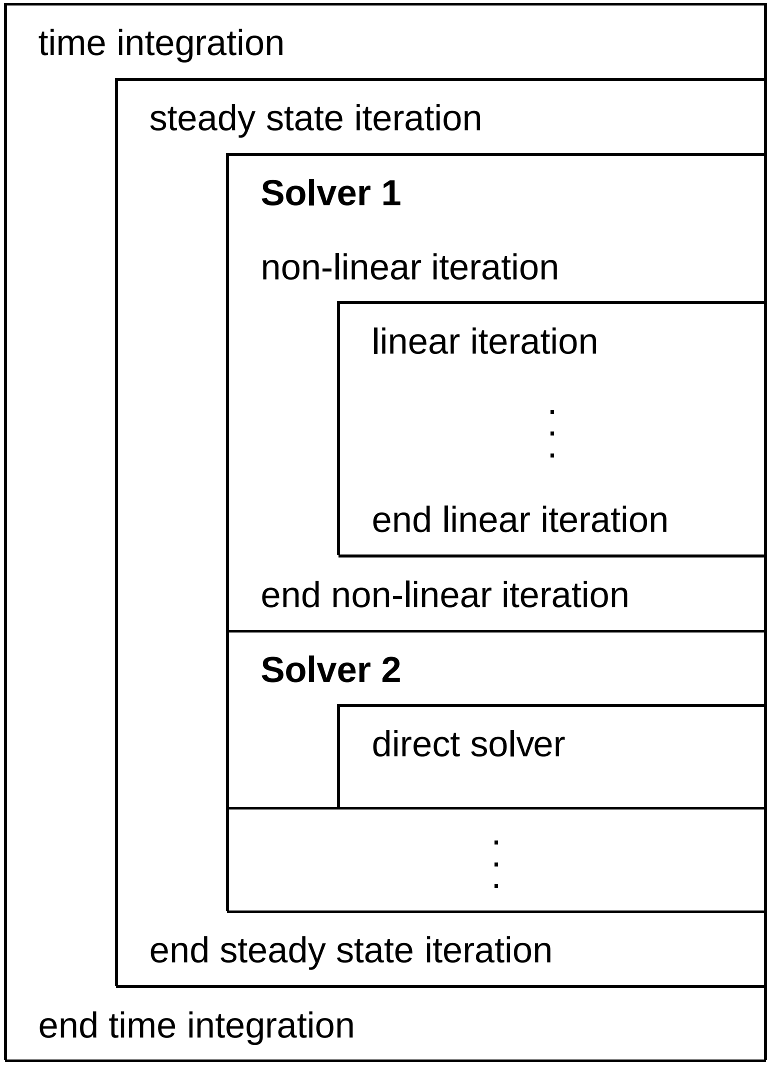
`Timestep Intervals`

Solution Levels



1. `Timestep Intervals`
2. `Steady State Max Iterations`
3.
4.
4.
3.
2. `Steady State Convergence Tolerance`
1.

Solution Levels



1. `Timestep Intervals`
2. `Steady State Max Iterations`
3. `Nonlinear Max Iterations`
4.
4.
3. `Nonlinear System Convergence Tolerance`
2. `Steady State Convergence Tolerance`
1.

Solution Levels

time integration	1. `Timestep Intervals`
steady state iteration	2. `Steady State Max Iterations`
Solver 1	
non-linear iteration	3. `Nonlinear Max Iterations`
linear iteration	4. `Linear System Max Iterations`
⋮	
end linear iteration	4. `Linear System Convergence Tolerance`
end non-linear iteration	3. `Nonlinear System Convergence Tolerance`
Solver 2	
direct solver	
⋮	
end steady state iteration	2. `Steady State Convergence Tolerance`
end time integration	1.

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Time integration

Two different schemes (`Timestepping Method =`):

- Crank-Nicolson method (`Crank-Nicolson`)
- Backward Differences Formulae (`BDF`)
  - `BDF Order`

Time integration

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- Crank-Nicolson method (`Crank-Nicolson`)
- Backward Differences Formulae (`BDF`)
  - `BDF Order`
- `Time Derivative Order` if 2, then Bossak method (`Bossak Alpha Real [-0.05]`)
- `Timestep Intervals`
- `Timestep Sizes`
- Adaptive timestepping only for BDF 1

Steady State Problem

Convergence between mutually dependent solver
(e.g., advective heat transfer = Navier-Stokes + Heat Transfer Euqation)

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- `Steady State Convergence Tolerance`

  $$||\mathbf{\Psi}^{(j)} - \mathbf{\Psi}^{(j-1)}||/||\mathbf{\Psi}^{(j)}|| < \epsilon_{\text{std}}$$

- `Steady State Max Iterations`

  $$j \leq j_{\text{max}}$$

- `Steady State Relaxation Factor`

  $$\lambda : \quad \mathbf{\Psi}^{(j)} \rightarrow \lambda\, \mathbf{\Psi}^{(j)} + (1-\lambda)\, \mathbf{\Psi}^{(j-1)}$$

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Non-linear Problem

Linearization:

$$\mathbf{A}(\mathbf{\Psi})\,\mathbf{\Psi} = \mathbf{f}(\mathbf{\Psi}) \quad \Rightarrow \quad \mathbf{A}(\mathbf{\Psi}^{(i-1)})\,\mathbf{\Psi}^{(i)} = \mathbf{f}(\mathbf{\Psi}^{(i-1)})$$

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Navier-Stokes:

- default *Picard iteration*: $\mathbf{u}\cdot\nabla\mathbf{u} \approx \mathbf{u}^{(i-1)}\cdot\nabla\mathbf{u}^{(i-1)}$

- `Nonlinear System Newton After Iterations` *Newton iteration*:

  $\mathbf{u}\cdot\nabla\mathbf{u} \approx \mathbf{u}^{(i)}\cdot\nabla\mathbf{u}^{(i-1)} + \mathbf{u}^{(i-1)}\cdot\nabla\mathbf{u}^{(i)} - \mathbf{u}^{(i-1)}\cdot\nabla\mathbf{u}^{(i-1)}$

Linear Solver

- Three solution methods for $\mathbf{A} \cdot \mathbf{\Psi} = \mathbf{f}$

```
Linear System Solver =
```

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  `Linear System Solver =`

- Direct methods (Keyword: `Direct`)

  `Linear System Direct Method =`

  - standard LAPACK (`banded`)
  - alternatively UMFPACK - Unsymmetrical Multi Frontal (`UMFPACK`)

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- Krylov subspace iterative methods (Keyword: `Iterative`)

  - `Linear System Iterative Method =`

    Conjugate Gradient (`CG`), Conjugate Gradient Squared (`CGS`), BiConjugate Gradient Stabilized (`BiCGStab`), Transpose-Free Quasi-Minimal Residual (`TFQMR`), Generalized Minimal Residual (`GMRES`)
  - `Linear System Convergence Tolerance` $||\mathbf{A}\,\mathbf{\Psi} = \mathbf{f}||/||\mathbf{f}|| < \epsilon_{\text{lin}}$

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  - `Linear System Convergence Tolerance` $||\mathbf{A}\,\mathbf{\Psi} = \mathbf{f}||/||\mathbf{f}|| < \epsilon_{\text{lin}}$

- Multilevel (Keyword: `Multigrid`) Geometric (GMG) and Algebraic (AMG) Multigrid

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Preconditioning

$$\mathbf{A} \cdot \mathbf{\Psi} = \mathbf{f}$$

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solving instead:

$$\mathbf{A}\mathrm{M}^{-1} \cdot \mathbf{\Phi} = \mathbf{f}\text{, with } \Phi = \mathbf{M} \cdot \mathbf{\Psi}$$

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Why?: $\mathbf{A}\mathrm{M}^{-1}$ shall have a towards convergence of iterative methods improved spectrum

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solving instead:

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Why?: $\mathbf{A}\mathbf{M}^{-1}$ shall have a towards convergence of iterative methods improved spectrum `Linear System Preconditioning =`

- `None`
- `Diagonal`
- `ILU`*n* $n = 0,1,2,\ldots$
- `ILUT`
- `Multigrid`