

Elmer Tutorials

CSC – IT Center for Science

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About Elmer Tutorials

The Elmer Tutorials is part of the documentation of Elmer finite element software. Elmer Tutorials gives examples on the use of Elmer in different field of continuum physics. Also coupled problems are included.

The present manual corresponds to Elmer software version 6.0 for Windows NT and Unix platforms. Latest documentations and program versions of Elmer are available (or links are provided) at <http://www.csc.fi/elmer>.

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Part I

ElmerGUI Problems

Tutorial 1

Heat equation – Temperature field of an L-shaped domain

Directory: TemperatureAngleGUI

Solvers: HeatSolve

Tools: ElmerGUI

Dimensions: 2D, Steady-state

Problem description

An L-shaped structure (see figure 27.1) is heated by an internal heat source, which magnitude is 1 W/m^3 . The density of the structure is 1 kg/m^3 and the heat conductivity is 1 W/mK . All the boundaries Γ_i are kept on constant temperature of 0 K . The problem is to solve the temperature distribution in the structure. Mathematically the problem to be solved is

$$\begin{cases} -\kappa\Delta T & = \rho f & \text{in}\Omega \\ T & = 0 & \text{on}\Gamma \end{cases} \quad (1.1)$$

where κ is the heat conductivity, T is the temperature and f is the heat source. It is assumed that density and heat conductivity are constants.

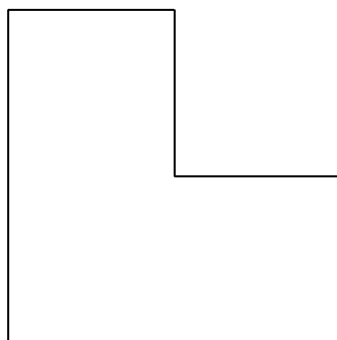


Figure 1.1: L-shaped domain

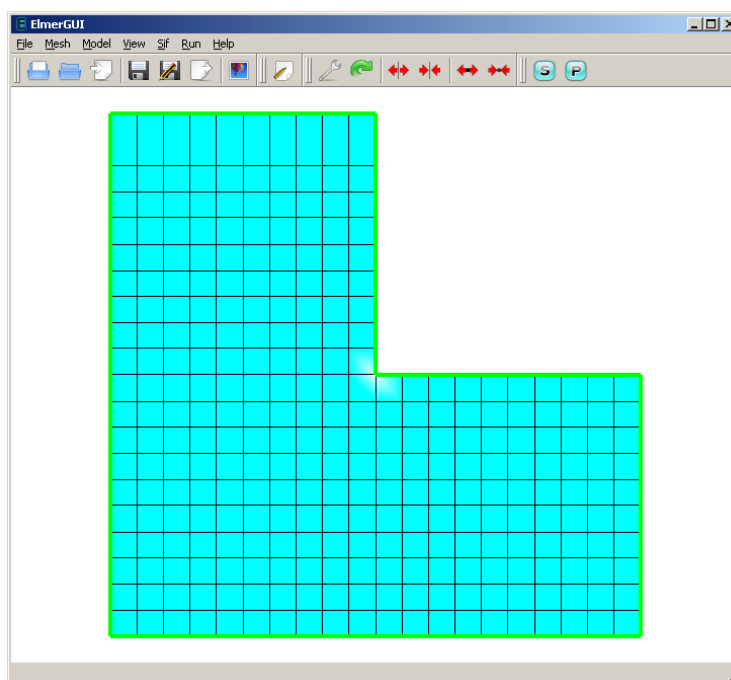


Figure 1.2: The finite element mesh in ElmerGUI

Solution procedure

Start ElmerGUI from command line or by clicking the icon in your desktop. Here we describe the essential steps in the ElmerGUI by writing out the clicking procedure. Tabulation generally means that the selections are done within the window chosen at the higher level.

The mesh is given in ElmerGrid format in file `angle.grd` in the samples directory of ElmerGUI, load this file.

```
File
  Open -> angle.grd
```

You should obtain your mesh and may check in the `Model summary` window that it consists of 341 nodes and 300 bilinear elements. If the mesh was successfully imported your window should look something in figure 1.2.

After we have the mesh we start to go through the `Model` menu from the top to bottom. In the `Setup` we choose things related to the whole simulation such as file names, time stepping, constants etc. The simulation is carried out in 2-dimensional cartesian coordinates and in steady-state. Only one steady-state iteration is needed as the case is linear.

```
Model
  Setup
    Simulation Type = Steady state
    Steady state max. iter = 1
```

Choose `Accept` to close the window.

In the equation section we choose the relevant equations and parameters related to their solution. In this case we'll have one set only one equation – the heat equation.

When defining Equations and Materials it is possible to assign the to bodies immediately, or to use mouse selection to assign them later. In this case we have just one body and one boundary and therefore its easier to assign the Equation and Material to it directly.

For the linear system solvers we are happy to use the defaults. One may however, try out different preconditioners (ILU1,...) or direct Umfpack solver, for example.

```
Model
Equation
  Add
    Name = Heat Equation
    Apply to bodies = 1
    Heat Equation
    Active = on
Apply
OK
```

The Material section includes all the material parameters. They are divided to generic parameters which are direct properties of the material without making any assumptions on the physical model, such as the mass. Other properties assume a physical law, such heat conductivity.

```
Model
Material
  Add
    Name = Ideal
    Apply to bodies = 1
    General
      Density = 1.0
    Heat Equation
      Heat Conductivity = 1.0
Apply
OK
```

A Body Force represents the right-hand-side of a equation that in this case represents the heat source.

```
Model
Body Force
  Add
    Name = Heating
    Heat Source = 1.0
    Apply to bodies = 1
Apply
OK
```

No initial conditions are required in steady state case.

In this case we have only one boundary and set it to zero.

```
Model
BoundaryCondition
  Add
    Heat Equation
      Temperature = 0.0
    Name = Zero
    Apply to boundaries = 1
Apply
OK
```

For the execution ElmerSolver needs the mesh files and the command file. We have know basically defined all the information for ElmerGUI to write the command file. After writing it we may also visually inspect the command file.

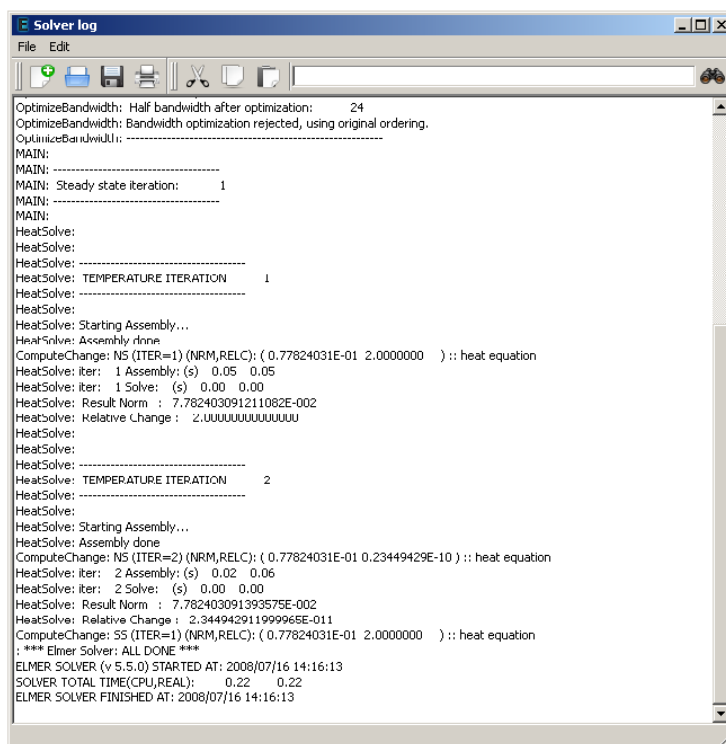


Figure 1.3: The output log of ElmerSolver when used under ElmerGUI

```

Sif
  Generate
  Edit -> look how your command file came out

```

Before we can execute the solver we should save the files in a directory. In saving the project all the necessary files for restarting the case will be saved to the destination directory.

```

File
  Save Project

```

After we have successfully saved the files we may start the solver

```

Run
  Start solver

```

A convergence view automatically pops up showing relative changes of each iteration. As the case is linear only one iteration was required for the solution and the second one just is needed to check the convergence. The resulting output log is shown in figure 1.3.

Note: if you face problems in the solution phase and need to edit the setting, always remember to save the project before execution.

To view the results we may start the the internal VTK widget, or use ElmerPost for postprocessing as is done here

```

Run
  Start postprocessor

```

The default configuration shows the surface mesh colored with temperature as in picture 1.4. You may test the different options. For example, choosing in the Color Mesh Edit window the Mesh Style to be Line and Edge Style to be All and pressing Apply the wireframe of the computational mesh is seen.

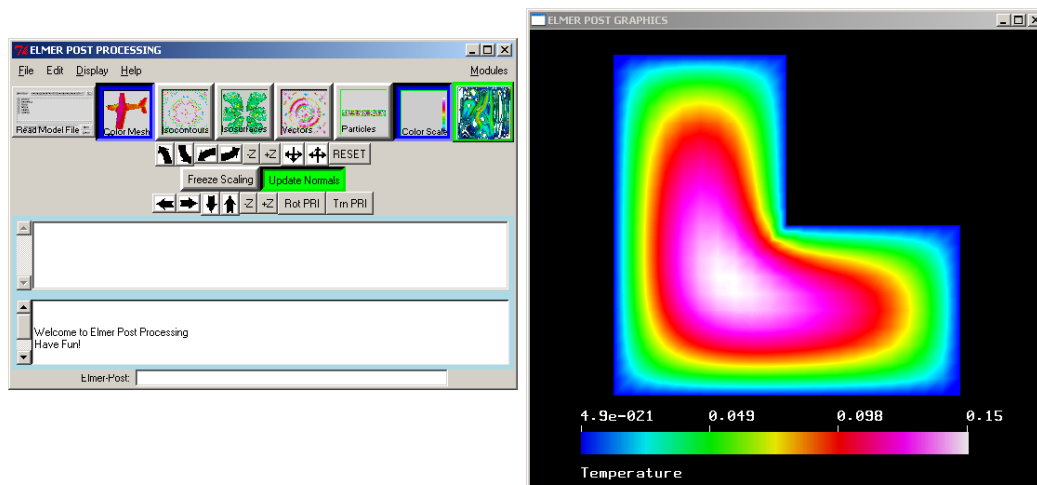


Figure 1.4: The temperature distribution of the L-shaped domain as visualized ElmerPost

Results

As a reference result the maximum temperature in the structure is given. For a comparison the same problem was solved six times with different element sizes using the `-relh` flag of ElmerGrid format located in the Mesh menu under Configure. After choosing Remesh and saving the mesh the solver may be recalled with the modified mesh.

The maximum temperature obtained by using different meshes is recorded in Table 1.1. From the results one can see that the result converges. With a denser mesh the result is more accurate, but solving the problem takes more calculation time. For reference, the CPU-time used in each case is also shown in the table (using Lenovo 60 laptop).

Table 1.1: Results with different element sizes

Model mesh h [m]	Elements	$\max(T)$ [K]	cpu time [s]
0.2	75	0.1452	0.14
0.1	300	0.1477	0.19
0.05	1200	0.1489	0.41
0.025	5120	0.14925	1.22
0.01	30800	0.14937	7.77

Tutorial 2

Heat equation – Temperature field of a solid object

Directory: TemperatureGenericGUI

Solvers: HeatSolve

Tools: ElmerGUI,tetgen,OpenCascade

Dimensions: 3D, Steady-state

Problem description

This tutorial tried to demonstrate how to solve the heat equation for a generic 3D object. The solid object (see figure 2.1) is heated internally by a heat source. At some part of the boundary the temperature is fixed. Mathematically the problem is described by the Poisson equation

$$\begin{cases} -\kappa\Delta T & = \rho f & \text{in } \Omega \\ T & = 0 & \text{on } \Gamma \end{cases} \quad (2.1)$$

where κ is the heat conductivity, T is the temperature and f is the heat source. It is assumed that density and heat conductivity are constants.

To determine the problem we assume that the part of the boundary is fixed at $T_0 = 293$ K, the internal heat generation is, $h = 0.01$ W/kg, and use the material properties of aluminium.

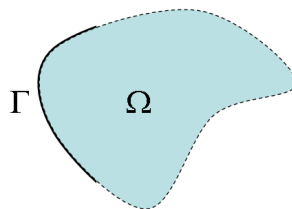


Figure 2.1: Generic object being heated

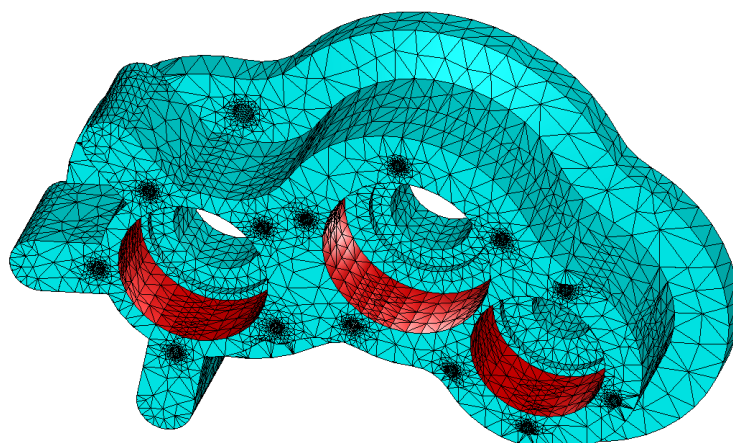


Figure 2.2: The computational mesh showing the three joined boundaries

Solution procedure

Start `ElmerGUI` from command line or by clicking the icon in your desktop. Here we describe the essential steps in the `ElmerGUI` by writing out the clicking procedure. Tabulation generally means that the selections are done within the window chosen at the higher level.

The geometry is given in step format in file `pump_carter_sup.stp` in the `samples/step` directory of `ElmerGUI`. This file is kindly provided at the AIM@SHAPE Shape Repository by INRIA. The heat equation is ideally suited for the finite element method and the solution may be found even at meshes that for some other problems would not be feasible. Therefore you may easily experiment solving the same problem with different meshes. If you lack either `OpenCascade` or `tetgen` you might try to solve the problem with the `grd` files `angle3d.grd`, `angles3d.grd`, `bench.grd`, or `cooler.grd`, for example.

The CAD geometry defined by the step file is transformed on-the-fly by `OpenCascade` library into a stl file for which `tetgen` (`tetlib`) creates tetrahedral volume discretization.

Load the input file:

File

Open -> `pump_carter_sup.stp`

You should obtain your mesh and may check in the number of element in the `Model` summary. With `tetgen` the default setting generate 33902 nodes and 110010 tetrahedral elements. In order to affect the mesh density study the command-line options in the `tetgen` manual. Here we continue with the default mesh.

The stl description of the mesh only provides with one body and one surface. Therefore we need to divide the existing surface. First choose the surface by clicking on it (it should turn red) and then

Mesh

Divide Surface

Divide

You may as well use the red icon with arrows pointing to different directions. The algorithm separates all boundaries with more than 20 angle separation between surfaces. We want to set the temperature at the inside of the holes and in that aim again join three boundaries (see figure 2.2). For that aim choose the three boundaries as shown in the picture by pressing the `Ctrl`-key down.

Mesh

Unify Surface

After we have the mesh we start to go through the `Model` menu from the top to bottom. In the `Setup` we choose things related to the whole simulation such as file names, time stepping, constants etc. The simulation

is carried out in 3-dimensional cartesian coordinates and in steady-state. Only one steady-state iteration is needed as the case is linear.

```
Model
  Setup
    Simulation Type = Steady state
    Steady state max. iter = 1
```

Choose **Accept** to close the window.

In the equation section we choose the relevant equations and parameters related to their solution. In this case we'll have one set only one equation – the heat equation.

When defining Equations and Materials it is possible to assign the to bodies immediately, or to use mouse selection to assign them later. In this case we have just one body and therefore its easier to assign the Equation and Material to it directly, whereas the active boundary is chosen graphically.

For the linear system solvers we are happy to use the defaults. One may however, try out different preconditioners (ILU1,..), for example.

```
Model
  Equation
    Add
      Name = Heat Equation
      Apply to bodies = Body 0
      Heat Equation
        Active = on
    Add
    OK
```

The Material section includes all the material parameters. They are divided to generic parameters which are direct properties of the material without making any assumptions on the physical model, such as the mass. Other properties assume a physical law, such heat conductivity. We choose Aluminium from the Material library which automatically sets for the needed material properties.

```
Model
  Material
    Add
      Material library
        Aluminium
      Apply to bodies = Body 0
    Add
    OK
```

A Body Force represents the right-hand-side of a equation that in this case represents the heat source.

```
Model
  Body Force
    Add
      Name = Heating
      Heat Source = 0.01
      Apply to bodies = Body 0
    Add
    OK
```

No initial conditions are required in steady state case.

In this case we have only one boundary and set it to room temperature. First we create the boundary condition

```
Model
  BoundaryCondition
```

```
Add
  Heat Equation
    Temperature = 293.0
  Name = RoomTemp
Add
OK
```

Then we set the boundary properties

```
Model
  Set boundary properties
```

Choose the defined group of three boundaries by clicking with the mouse and apply the condition for this boundary.

```
Boundary condition
  RoomTemp
```

For the execution ElmerSolver needs the mesh files and the command file. We have know basically defined all the information for ElmerGUI to write the command file. After writing it we may also visually inspect the command file.

```
Sif
  Generate
  Edit -> look how your command file came out
```

Before we can execute the solver we should save the files in a directory. In saving the project all the necessary files for restarting the case will be saved to the destination directory.

```
File
  Save Project
```

After we have successfully saved the files we may start the solver

```
Run
  Start solver
```

A convergence view automatically pops up showing relative changes of each iteration. As the case is linear only one iteration was required for the solution and the second one just is needed to check the convergence. The norm of the solution should be around 389.853 K.

Note: if you face problems in the solution phase and need to edit the setting, always remember to regenerate the sif file and save the project before execution.

Postprocessing

To view the results we may use the ElmerPost postprocessor or start the the internal VTK widget as is done here,

```
Run
  Postprocessor (VTK)
```

The default configuration shows just the object. To color the surface with the temperature choose

```
Surfaces
  Surface: Temperature
  Apply
```

There seems to be some hanging nodes generated by tetgen with zero temperature. Thus, fixing the Min to 293 and setting crossing the `Keep limits` box gives better color scale. You may also turn on opacity in order to see through the object, 10-20% is a good value. This way you'll able to see some isosurfaces that you might want to define. Some examples of the visualizations may be seen in figure 2.3.

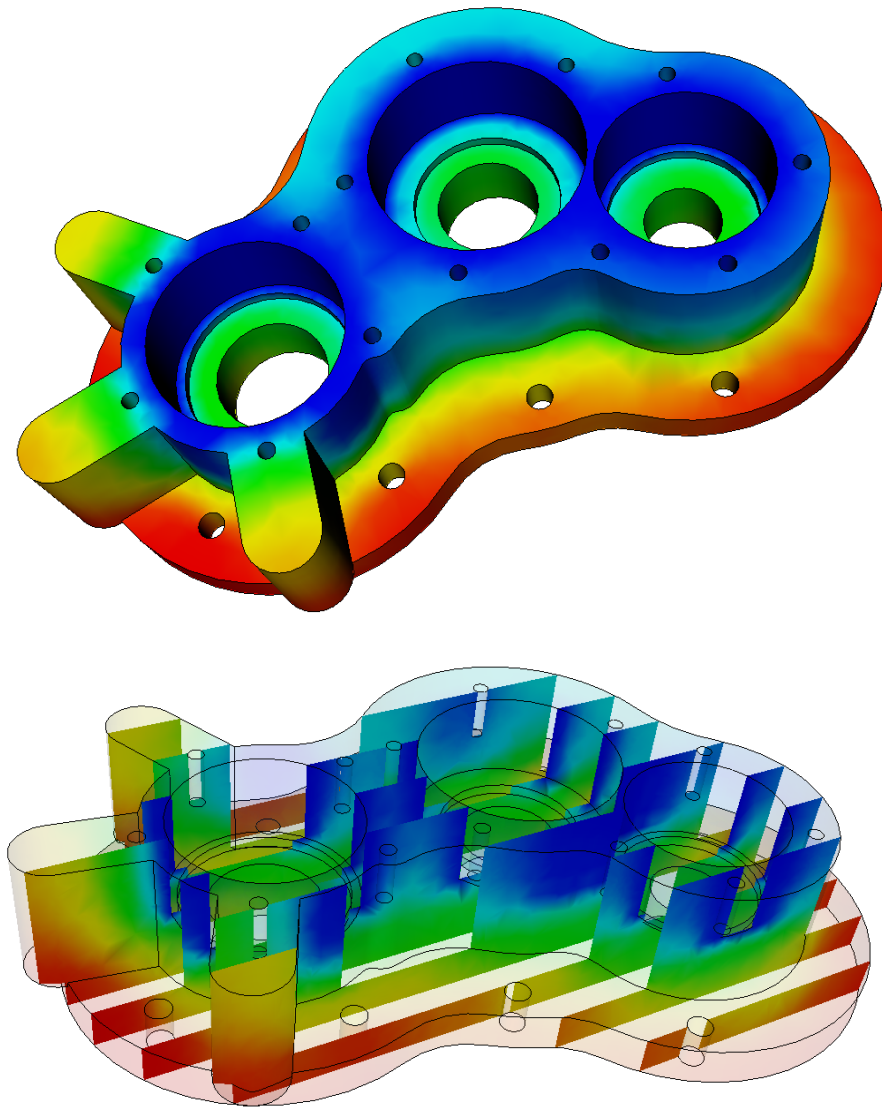


Figure 2.3: The temperature distribution of the solid object domain as visualized using the VTK-based postprocessor

Tutorial 3

Linear elasticity equation – Loaded elastic beam

Directory: ElasticBeam3D

Solvers: StressSolve

Tools: ElmerGUI

Dimensions: 3D, Steady-state

Case definition

Assume a homogenous, elastic beam being rigidly supported on one end. On the other end it is subjected with a load of 2000 N resulting from an attached object in the gravitational field. The gravity affects also the beam itself. The length of the beam is 1 m and the thickness is 0.05 m, and the width 0.1 m. Material properties of the beam are those of dry pine timber: Poisson ratio 0.37, Young's modulus $10 \cdot 10^9 \text{N/m}^2$, and density 550kg/m^3 . The problem is to solve the displacement and stress field of the beam. Here the `StressSolve` routine based on the linear theory of elasticity is applied.

Solution procedure

The mesh is given in ElmerGrid format in file `beam3d.grd`, load this file.

File

Open -> `beam3d.grd`

You should obtain your mesh and may check that it consists of 6073 nodes and of 1200 quadratic hexahedral elements. The second order elements give improved accuracy compared to the first order elements as they avoid the phenomenon known as locking.

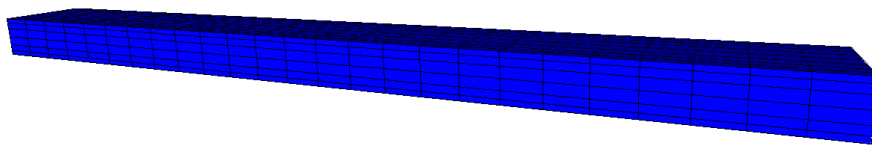


Figure 3.1: The mesh used in the computations

After we have the mesh we start to go through the Model menu from the top to bottom. In the Setup we choose things related to the whole simulation such as file names, time stepping, constants etc. The simulation is carried in steady-state in 3-dimensional cartesian coordinates.

```
Model
  Setup
    Simulation Type = Steady state
    Steady state max. iter = 1
```

In the Equation section we choose the relevant equations which in this case only includes the Linear elasticity equation which solves the problem according to linear elastic theory. We also want to compute the stresses as a post-processing step. For the linear system solvers we change the maximum number of iterations and the preconditioner since the default settings do not lead to satisfactory convergence in this case.

```
Model
  Equation
    Name = Elasticity
    Apply to Bodies = Body 1
    Linear elasticity
      Active = on
      Calculate Stresses = on
    Edit Solver Setting
      Linear System
        Max. iterations = 1000
        Preconditioning = ILU1
    Apply
  Add
  OK
```

The Material section includes all the material parameters. They are divided to generic parameters which are direct properties of the material without making any assumptions on the physical model, such as the mass. Other properties assume a physical law, such as Young's modulus and Poisson ratio.

```
Model
  Material
    Name = Pine
    General
      Density = 550
    Linear Elasticity
      Youngs Modulus = 10.0e9
      Poisson ratio = 0.37
    Apply to Bodies = Body 1
  Add
  OK
```

In this case there is a body force i.e. the gravity acting on the beam. We assume that the gravity points to the negative y direction.

```
Model
  BodyForce
    Name = Gravity
    Linear Elasticity
      Force 2 = $ -9.81 * 550
    Apply to Bodies = Body 1
  Add
  OK
```

Here we use a MATC expression for computing the volume force. This expression is constant and is computed when the command file is interpreted.

Convergence should be obtained with the default initial condition i.e. zero for all fields, hence no initial condition is applied.

The first boundary condition fixes the beam rigidly at the wall. The second boundary condition distributes the load of 2000 N uniformly on the area of $5.0e-3 \text{ m}^2$.

Model

```
BoundaryCondition
  Name = Wall
  Linear elasticity
    Displacement 1 = 0.0
    Displacement 2 = 0.0
    Displacement 3 = 0.0
  Add
  New

  Name = Mass
  Linear elasticity
    Force 2 = -4.0e5
  Add
```

The conditions may also be assigned to boundaries in the Boundary condition menu, or by clicking with the mouse. Here we use the latter approach as that spares us of the need to know the indexes of each boundary.

Model

```
Set boundary properties
  Choose the wall end of the beam -> set boundary condition Wall
  Choose the other end of the beam -> set boundary condition Mass
```

For the execution ElmerSolver needs the mesh files and the command file. We have know basically defined all the information for ElmerGUI to write the command file. After writing it we may also visually inspect the command file.

Sif

```
Generate
Edit -> look how your command file came out
```

Before we can execute the solver we should save the files in a directory. The project includes all the files needed to restart the case.

File

```
Save Project
```

After we have successfully saved the files we may start the solver

Run

```
Start solver
```

A convergence view automatically pops up showing relative changes of each iteration. Two iterations are performed, the second one only to ensure convergence at the nonlinear level.

Results

When there are some results to view we may start the postprocessor, this time we use ElmerPost.

Run

```
Start postprocessor
```

As a result the absolute value of maximum displacement is shown. The maximum displacement is 6.36 cm. To visualize the displacement in the geometry using ElmerPost can be done with the following command in the Elmer-Post command line.

```
math n0=nodes
math nodes=n0+Displacement
```

To redraw the picture with new settings use the rightmost icon on the top row. The resulting picture is shown in Fig 3.2. Note that the displacement are so large that the assumption of linearity may be severely

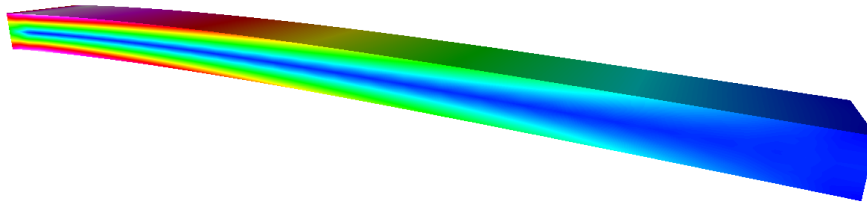


Figure 3.2: The displaced shape of the elastic beam colored with the von Mises stresses

questioned. When further increasing the loading one should resort to a solver that is able to catch the geometric nonlinearities.

Extra task: Gravity in x direction

The beam should be more rigid if the beam is oriented differently. For that aim, change the direction of gravity to orient in the negative x . Change the body force

```
Model
  BodyForce
    Linear Elasticity
      Force 1 = $ -9.81*550
    Update
  OK
```

and the boundary condition

```
Model
  BoundaryCondition
    Linear elasticity
      Force 1 = -4.0e5
    Update
  OK
```

The rigidity should scale as dh^3 and hence the maximum displacement should be reduced roughly to one quarter of the original.

Tutorial 4

Navier-Stokes equation – Laminar incompressible flow passing a step

Directory: FlowStepGUI

Solvers: FlowSolve

Tools: ElmerGUI

Dimensions: 2D, Steady-state

Case definition

This tutorial represents the canonical step flow of viscous fluid. A fluid, flowing past a step (see figure 29.1), has the density 1 kg/m and viscosity 0.01 kg/ms. The velocity profile at the inlet is parabolic with a mean velocity $\langle v_x \rangle = 1.0$ m/s and $v_y = 0.0$ m/s. At the outlet only the vertical component is defined, $v_y = 0.0$ m/s. At all other walls the no-slip boundary condition, $\vec{v} = 0$, is applied. Thus the Reynolds number for the case is around 100.



Figure 4.1: Geometry of the step flow problem

Mathematically the problem to be solved is

$$\begin{cases} -\nabla \cdot (2\mu\bar{\varepsilon}) + \rho\vec{u} \cdot \nabla\vec{u} + \nabla p = 0 & \text{in } \Omega \\ \nabla \cdot \vec{u} = 0 & \text{in } \Omega \end{cases} \quad (4.1)$$

with the boundary conditions

$$\begin{cases} u_x = 1 & \text{on } \Gamma_{inlet} \\ u_x = 0 & \text{on } \Gamma_{no-slip} \\ u_y = 0 & \text{on } \Gamma_{inlet} \cup \Gamma_{outlet} \cup \Gamma_{no-slip} \end{cases} \quad (4.2)$$

where μ is the viscosity, $\bar{\varepsilon}$ is the strain tensor, ρ is the density, \vec{u} is the velocity and p is the pressure. It is assumed that the density and viscosity are constants.

Solution procedure

The mesh is given in ElmerGrid format in file `step.grd`, load this file.

File

```
Open -> step.grd
```

You should obtain your mesh and may check that it consists of 9696 nodes and of 9442 bilinear elements.

Model

```
Summary...
```

After we have the mesh we start to go through the Model menu from the top to bottom. In the Setup we choose things related to the whole simulation. The steady-state simulation is carried out in 2-dimensional cartesian coordinates, which are also the defaults.

Model

```
Setup
```

```
Simulation Type = Steady state
```

```
Coordinate system = Cartesian
```

In the equation section we choose the relevant equations and parameters related to their solution. In this case the only the Navier-Stokes equation is needed.

When defining Equations and Materials it is possible to assign the to bodies immediately, or to use mouse selection to assign them later. In this case we have just one body and therefore its easier to assign the Equation and Material to it directly. One could also edit the solver setting in order to try different strategies for solving the nonlinear or linear system. Initially the Navier-Stokes solver uses the more robust Picard iteration which is changed to Newton iteration after few initial steps. For the given viscosity the default values are ok, but may need tuning when going into higher Reynolds numbers.

Model

```
Equation
```

```
Name = Navier-Stokes
```

```
Apply to Bodies = 1
```

```
Navier-Stokes
```

```
Active = on
```

```
Edit Solver Setting
```

```
Nonlinear System
```

```
Max. iterations = 20
```

```
Newton after iterations = 3
```

```
Add
```

```
OK
```

The Material section includes all the material parameters. They are divided to generic parameters which are direct properties of the material without making any assumptions on the physical model, such as the density. Other properties assume a physical law, such as viscosity.

Model

```
Material
```

```
Name = Ideal
```

```
General
```

```
Density = 1.0
```

```
Navier-Stokes
```

```
Viscosity = 0.01
```

```
Apply to Bodies = 1
```

```
Add
```

```
OK
```

The current case does not have any body forces. Convergence should also be obtained using the default initial condition which sets all field values to zero. Hence no setting for initial condition are needed.

Only one boundary condition may be applied to each boundary and therefore all the different physical BCs for a boundary should be grouped together. In this case the Temperature and Velocity. The side walls are assumed to be adiabatic.

The parabolic inlet-profile is achieved using the MATC environment. To be able to edit the content of the inlet profile click `Enter` to open an edit box for the `Velocity 1`. The given expression will be interpreted at run-time so that $v_x = 6(y - 1)(2 - y)$. As $y \in [1, 2]$ thereby creating a parabolic velocity profile with a mean velocity of unity.

Model

```
BoundaryCondition
  Name = Inlet
  Navier-Stokes
    Velocity 1 = Variable Coordinate 2; Real MATC ``6*(tx-1)*(2-tx)``
    Velocity 2 = 0.0
  Add
  New

  Name = Outlet
  Navier-Stokes
    Velocity 2 = 0.0
  Add
  New

  Name = Walls
  Navier-Stokes
    Velocity 1 = 0.0
    Velocity 2 = 0.0
  Add
```

The conditions may also be assigned to boundaries in the Boundary condition menu, or by clicking with the mouse. Here we use the latter approach as that spares us of the need to know the indexes of each boundary.

Model

```
Set boundary properties
  Choose Inlet -> set boundary condition Inlet
  Choose Outlet -> set boundary condition Outlet
  Choose Walls -> set boundary condition Walls
```

For the execution ElmerSolver needs the mesh files and the command file. We have now basically defined all the information for ElmerGUI to write the command file. After writing it we may also visually inspect the command file.

Sif

```
Generate
Edit -> look how your command file came out
```

Before we can execute the solver we should save the files in a directory. The project includes all the files needed to restart the case. Create a suitable directory for the case if needed.

File

```
Save Project
```

After we have successfully saved the files we may start the solver

Run

```
Start solver
```

A convergence view automatically pops up showing relative changes of each iteration. The problem should converge in nine iterations. When there are some results to view we may start the postprocessor also

```
Run
  Start postprocessor
```

Results

The results may be viewed using the postprocessor as shown in Figure 4.2 and 4.3. One may also register specific values, for example the pressure difference is 4.23 Pa, the minimum and maximum lateral velocities are -0.164 m/s and 1.3709 m/s, respectively. One special result of interest is the point, on the x-axis, at which the direction of the flow changes. In this case its position is about 6.6 m after the step.

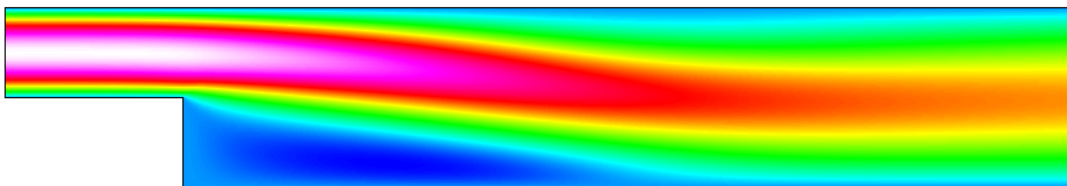


Figure 4.2: Absolute value of the velocity field



Figure 4.3: Pressure field

Extra task: Decreasing the viscosity

Try what happens if the viscosity is further decreased by a factor 10. Convergence may be difficult to obtain. Some tricks that may be tested include

- Introducing a relaxation factor (typically in the range 0.5-0.7)
- Increasing number of nonlinear iterations
- Favoring Picard iteration over Newton
- Increasing mesh density (and length of domain)

Don't be worried if you fail to find convergence. This task will mainly act as a motivator in using turbulence models for higher Reynolds numbers.

Remember to re-perform the following phases in order to get the updated results


```
Sif
  Generate
File
  Save Project
Run
  Start solver
```

You may just reload the results in the postprocessor rather than closing and opening the program.

Tutorial 5

Navier-Stokes equation – Turbulent incompressible flow passing a step

Directory: FlowStepGUI

Solvers: FlowSolve, KESolver

Tools: ElmerGUI

Dimensions: 2D, Steady-state

Case definition

This tutorial is a natural continuation of the tutorial 4 where the same case was solved with a smaller Reynolds number. It is advisable to study that case before.

When Reynolds number increases the Navier-Stokes equations do not possess any steady-state solution. Basically the solution can be averaged simulating the transient flow over time. However, the computational cost of this approach is often very heavy particularly while at high Reynolds numbers the computational mesh in direct numerical simulation needs to be very dense. Instead it is customary to solve time-averaged equations. Unfortunately these equations include unknown correlations between quantities that need to be modeled in some way.

The workhorse of turbulence modeling is the $k - \varepsilon$ model which is used in this tutorial. The $k - \varepsilon$ model is a two-equation model that introduces two additional variables – the turbulent kinetic energy k and the turbulent dissipation ε which determines the scale of the turbulence.

The case under study is the canonical step flow of viscous fluid. A fluid, flowing past a step has the density 1 kg/m^3 and viscosity $1.0e - 4 \text{ kg/ms}$. The velocity profile at the inlet is defined by a parabolic profile with mean velocity $v_x = 1.0 \text{ m/s}$ and $v_y = 0.0 \text{ m/s}$. This way the Reynolds number will be 10000. At the outlet only the vertical component is defined, $v_y = 0.0 \text{ m/s}$. At all other walls the no-slip boundary condition, $\vec{v} = 0$, is applied.

Also the new turbulent variables require boundary conditions. In the inflow the condition could reflect the values for a developed turbulent profile. Here we roughly estimate the turbulent kinetic energy and elongate the distance before the step to have a fully developed turbulent profile. From the literature it is the turbulent intensity i.e. the kinetic energy of turbulence vs. the kinetic energy of mean flow scales as $0.16\text{Re}^{-1/8}$. For our current configuration a estimate for the turbulent kinetic energy is 0.00457. For the walls a no-slip condition is applied which also sets the values of the turbulent parameters accordingly. Also a boundary layer model could be used but here our mesh should be able to capture even the boundary phenomena quite well.

Solution procedure

The mesh is given in ElmerGrid format in file `steplong.grd`, load this file.

File

```
Open -> steplong.grd
```

You should obtain your mesh and may check that it consists of 14584 nodes and of 14175 bilinear elements.

```
Model
  Summary...
```

After we have the mesh we start to go through the Model menu from the top to bottom. In the Setup we choose things related to the whole simulation. The steady-state simulation is carried out in 2-dimensional cartesian coordinates, which are also the defaults. The coupled system converges unfortunately quite slowly and hence we need to increase the number of maximum iterations.

```
Model
  Setup
    Simulation Type = Steady state
    Coordinate system = Cartesian
    Steady state max. iter = 100
```

In the equation section we choose the relevant equations and parameters related to their solution. In this case the Navier-Stokes and $k - \varepsilon$ equations are needed. We want to solve the Navier-Stokes and $k - \varepsilon$ equations iteratively using only one nonlinear iteration for optimal convergence of the coupled system. Some relaxation is needed in order to achieve convergence at all. We also relax a little bit on the steady state convergence tolerance. Initially the Navier-Stokes solver uses the more robust Picard iteration which may be changed to Newton iteration after the iteration progresses. However, here we want to suppress the use of Newton linearization since it seems to cause problems with the $k - \varepsilon$ equation.

```
Model
  Equation
    Name = Flow equations
    Apply to Bodies = Body 1
    Navier-Stokes
      Active = on
      Edit Solver Setting
        Nonlinear System
          Max. iterations = 1
          Relaxation factor = 0.5
          Newton after tolerance = 0.0
        Steady state
          Convergence tol. = 1.0e-4
    K-Epsilon
      Active = on
      Edit Solver Setting
        Nonlinear System
          Max. iterations = 1
          Relaxation factor = 0.5
        Steady state
          Convergence tol. = 1.0e-4
  Add
  OK
```

The Material section includes all the material parameters. They are divided to generic parameters which are direct properties of the material without making any assumptions on the physical model, such as the density. Other properties assume a physical law, such as viscosity. For the model parameters of the turbulent equations we are happy with the defaults.

```
Model
  Material
    Name = Ideal
    General
```

```

Density = 1.0
Navier-Stokes
  Viscosity = 1.0e-4
  Viscosity Model = K-Epsilon
Apply to Bodies = Body 1
Add
OK

```

The current case does not have any body forces. To help in the convergence we make an rude initial guess.

```

Model
Initial Condition
Name = Initial Guess
Navier-Stokes
  Velocity 1 = 0.0
  Velocity 2 = 0.0
K-Epsilon
  Kinetic Energy = 0.00457
  Kinetic Dissipation = 1.0e-4

```

When defining Boundary conditions it is possible to assign the to boundaries immediately, or to use mouse selection to assign them later. In this case we have use the latter since we do not necessarily know the numbering of boundaries by heart. There is a special boundary condition that takes care of the Boundary conditions for the noslip walls for both the Navier-Stokes and $k - \varepsilon$ equation. Additionally there are inlet and outlet conditions. For the inlet click Enter to open an edit box for the Velocity 1 when typing in the expression, which will be evaluated at run-time so that $v_x = 6(y - 1)(2 - y)$.

```

Model
BoundaryCondition
Add
Name = Inlet
Navier-Stokes
  Velocity 1 = Variable Coordinate 2; Real MATC ``6*(tx-1)*(2-tx)``
  Velocity 2 = 0.0
K-Epsilon
  Kinetic Energy = 0.00457
  Kinetic Dissipation = 1.0e-4
Add
New

Name = Outlet
Navier-Stokes
  Velocity 2 = 0.0
Add
New

Name = Walls
Navier-Stokes
  Noslip Wall BC = on
Add

```

The conditions may also be assigned to boundaries in the Boundary condition menu, or by clicking with the mouse. Here we use the latter approach as that spares us of the need to know the indexes of each boundary.

```

Model

```

```
Set boundary properties
  Choose inlet -> set boundary condition Inlet
  Choose outlet -> set boundary condition Outlet
  Choose walls -> set boundary condition Walls
```

For the execution ElmerSolver needs the mesh files and the command file. We have now basically defined all the information for ElmerGUI to write the command file. After writing it we may also visually inspect the command file.

```
Sif
  Generate
  Edit -> look how your command file came out
```

Before we can execute the solver we should save the files in a directory. The project includes all the files needed to restart the case. Create a suitable directory for the case if needed.

```
File
  Save Project
```

After we have successfully saved the files we may start the solver

```
Run
  Start solver
```

A convergence view automatically pops up showing relative changes of each iteration. The convergence is far from monotonic but computation should terminate when sufficient convergence is reached after 30 iterations.

Results

When there are some results to view we may start the postprocessor. This time we use the internal VTK based postprocessor for visualization. Also ElmerPost could be used.

```
Run
  Postprocessor (VTK)
```

The results may be viewed using the postprocessor as shown in Figures 5.1 and 5.2. One may also register specific values, for example the pressure difference is 0.302 Pa, the minimum horizontal and vertical velocities are -0.213 m/s and -0.0834 m/s, respectively. One special result of interest is the point, on the x-axis, at which the direction of the flow changes. In this case its position is about 5.1 m after the step.

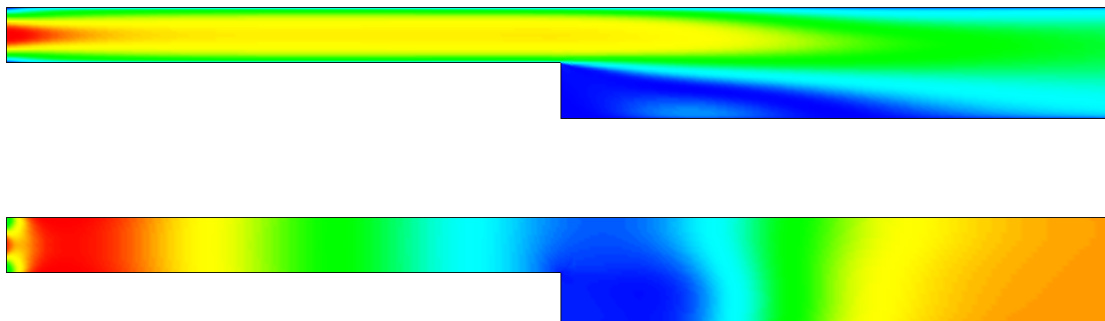


Figure 5.1: Variables of the Navier-Stokes solver: absolute velocity on top and pressure on bottom

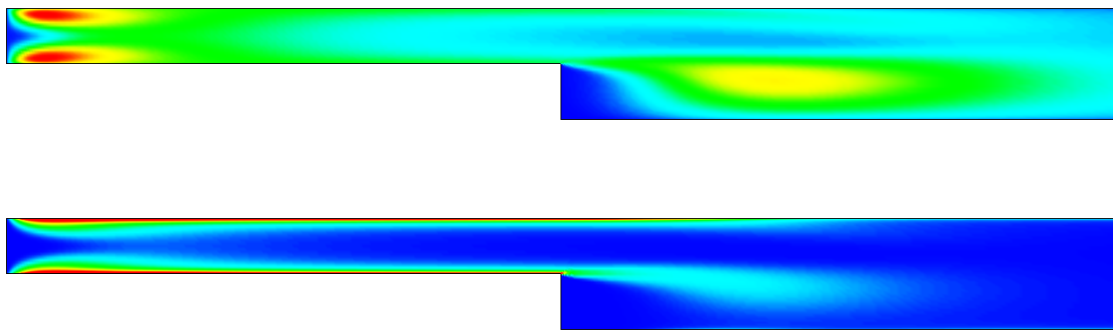


Figure 5.2: Variables of the $k - \varepsilon$ solver: kinetic energy on top and its dissipation on bottom

Tutorial 6

Transient flow and heat equations – Rayleigh-Benard instability

Directory: RayleighBenardGUI

Solvers: HeatSolve, FlowSolve

Tools: ElmerGUI

Dimensions: 2D, Transient

Case definition

This tutorial is about simulating the developing of the Rayleigh-Benard instability in a rectangular domain (Figure 30.1) of dimensions 0.01 m height and 0.06 m length. The simulation is performed with water and the material parameters of water required by the Elmer model are presented in Table 30.1. The temperature difference between the upper and lower boundary is set to 0.5 so that lower one has the temperature of 293.5 K and the upper one has the temperature of 293 K.

The density of water is inversely proportional to its temperature. Thus, heated water starts to flow upwards, and colder downwards due to gravity. In this case we assume that the Boussinesq approximation is valid for thermal incompressible fluid flow. In other words, the density of the term $\rho \vec{f}$ in the incompressible Navier-Stokes equation can be redefined by the Boussinesq approximation

$$\rho = \rho_0(1 - \beta(T - T_0))$$

where β is the heat expansion coefficient and the subscript 0 refers to a reference state.

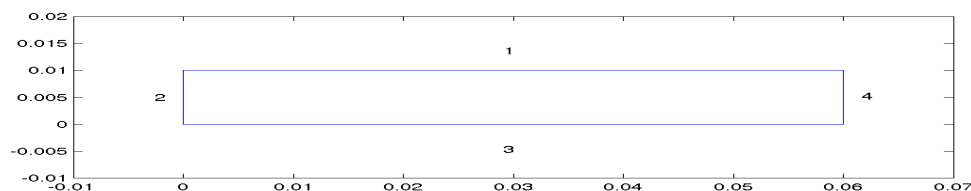


Figure 6.1: Domain.

Table 6.1: Material parameters for water

parameter	value
density	998.3 kg/m ³
viscosity	1040e-6 Ns/m ²
heat capacity	4183 J/(kg·K)
heat conductivity	0.58 W/(m·K)
heat expansion coefficient	2.07e-4 K ⁻¹
reference temperature	293 K

Solution procedure

The mesh is given in ElmerGrid format in file `box.grd`, load this file.

File

```
Open -> box.grd
```

You should obtain your mesh and may check that it consists of 3036 bilinear elements.

There is a possibility to divide and unify edges to simplify the case definition in the future.

Choose (left wall + right wall (Ctrl down)) -> unify edge

After we have the mesh we start to go through the Model menu from the top to bottom. In the Setup we choose things related to the whole simulation such as file names, time stepping, constants etc. The simulation is carried out in 2-dimensional cartesian coordinates. 2nd order bdf time-stepping method is selected with 200 steps and with step size of two seconds.

Model

Setup

```
Simulation Type = Transient
Steady state max. iter = 20
Time Stepping Method = bdf
BDF Order = 2
Time Step Intervals = 200
Time Step Sizes = 2.0
Gravity = ...
```

In the equation section we choose the relevant equations and parameters related to their solution. In this case we'll have one set of equations (named "Natural Convection") which consists of the heat equation and of the Navier-Stokes equation.

When defining Equations and Materials it is possible to assign the to bodies immediately, or to use mouse selection to assign them later. In this case we have just one body and therefore its easier to assign the Equation and Material to it directly. It is important to select the convection to be computed since that couples the velocity field to the heat equation.

The system may include nonlinear iterations of each equation and steady state iterations to obtain convergence of the coupled system. It is often a good idea to keep the number of nonlinear iterations in a coupled case low. Here we select just one nonlinear iteration for both equations. For the linear system solvers we are happy to use the defaults. One may however, try out different preconditioners (ILU1,...) or direct Umfpack solver, for example.

Model

Equation

```
Name = Natural Convection
Apply to Bodies = 1
Heat Equation
```



```

Active = on
Convection = Computed
Edit Solver Setting
  Nonlinear System
    Max. iterations = 1
Navier-Stokes
  Active = on
  Edit Solver Setting
    Nonlinear System
      Max. iterations = 1
Add
OK

```

The Material section includes all the material parameters. They are divided to generic parameters which are direct properties of the material without making any assumptions on the physical model, such as the mass. Other properties assume a physical law, such as conductivities and viscosity.

Here we choose water at room temperature from the material library. You may click through the material parameters of the various solvers to ensure that the properties are indeed as they should be. Any constant set of units may be used in Elmer. The natural choice is of course to perform the computations in SI units.

Apart from the properties from the material database, we reference temperature for the Boussinesq approximation.

```

Model
  Material
    Material library
      Water (room temperature)
    General
      Reference Temperature = 293
    Apply to Bodies = 1
  Add
  OK

```

A Body Force represents the right-hand-side of a equation. It is generally not a required field for a body. In this case, however, we apply the buoyancy resulting from heat expansion as a body force to the Navier-Stokes equation.

```

Model
  Body Force
    Name = Buoyancy
    Apply to Bodies = 1
    Navier-Stokes
      Boussinesq = on
  Add
  OK

```

Initial conditions should be given to transient cases. In this case we choose a constant Temperature field and an small initial velocity that initializes the symmetry break.

```

Model
  Initial Condition
    Name = Initial Guess
    Heat Equation
      Temperature = 293
    Navier-Stokes
      Velocity 1 = 1.0e-9
      Velocity 2 = 0.0

```

Only one boundary condition may be applied to each boundary and therefore all the different physical BCs for a boundary should be grouped together. In this case the Temperature and Velocity. The side walls are assumed to be adiabatic.

Model

```
BoundaryCondition
  Name = Bottom
  Heat Equation
    Temperature = 293.5
  Navier-Stokes
    Velocity 1 = 0.0
    Velocity 2 = 0.0
  Add
  New

  Name = Top
  Heat Equation
    Temperature = 293
  Navier-Stokes
    Velocity 1 = 0.0
    Velocity 2 = 0.0
  Add
  New

  Name = Sides
  Navier-Stokes
    Velocity 1 = 0.0
    Velocity 2 = 0.0
  Add
```

The conditions may also be assigned to boundaries in the Boundary condition menu, or by clicking with the mouse. Here we use the latter approach as that spares us of the need to know the indexes of each boundary.

Model

```
Set boundary properties
  Choose Bottom -> set boundary condition Bottom
  Choose Top -> set boundary condition Top
  Choose Sides -> set boundary condition Sides
```

For the execution ElmerSolver needs the mesh files and the command file. We have now basically defined all the information for ElmerGUI to write the command file. After writing it we may also visually inspect the command file.

Sif

```
Generate
Edit -> look how your command file came out
```

Before we can execute the solver we should save the files in a directory. The project includes all the files needed to restart the case.

File

```
Save Project
```

After we have successfully saved the files we may start the solver

Run

```
Start solver
```

A convergence view automatically pops up showing relative changes of each iteration. When there are some results to view we may start the postprocessor also

```
Run
  Start postprocessor
```

Results

Due to the number of the time-steps the simulation may take around ten minutes. You may inspect the results with ElmerPost as the time-steps are computed, or wait until all timesteps have been computed. When opening the result file using ElmerGUI ElmerPost only opens the first time-step. Therefore it is important to reopen the file and load the time-steps of interest. Pressing the button `All` selects all the calculated time steps. A video of the results can be viewed by selecting the option `Timestep Control` and pressing the button `Loop` under the `Edit` menu.

In Figures 30.2 and 30.3 the obtained temperature distribution and the velocity vectors are presented. The maximum velocity in the system should be about 0.5 mm/s.

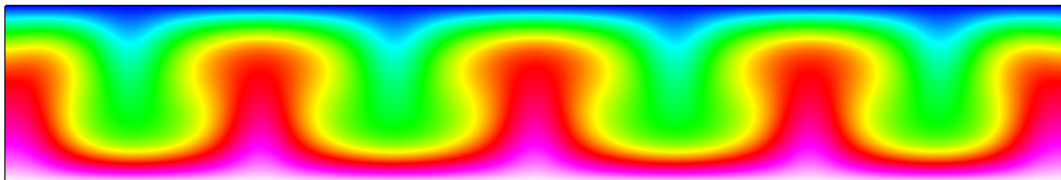


Figure 6.2: Temperature distribution at 260 s.

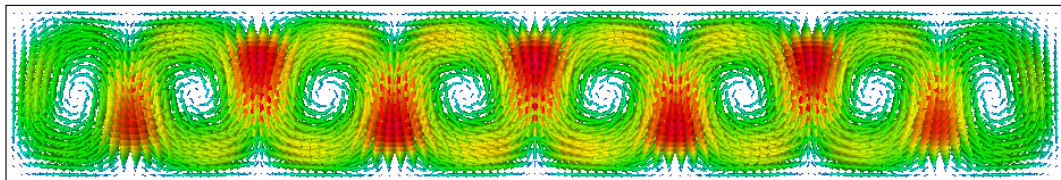


Figure 6.3: Velocity vectors at 260 s.

Extra task: Sensitivity to temperature difference

If you have time you may try to solve the case with different parameters. Changing the temperature difference is one way of affecting the instability of the system. Decreasing the temperature differences the system eventually becomes steady state and the convection rolls vanish altogether. Increasing the temperature difference may increase the number of convection rolls and eventually the system becomes fully chaotic. Note that changing the temperature difference also affects to the time scale of the wake.

Tutorial 7

Electrostatic equation – Computation of fringe capacitance

Directory: FringeCapacitance

Solvers: StatElecSolver

Tools: ElmerGUI

Dimensions: 2D, Steady-state

Case definition

This case presents solving the Laplace equation for electric potential.

$$-\nabla \cdot \varepsilon \nabla \phi = 0 \quad \in \Omega \quad (7.1)$$

where the electric potential ϕ is given at conducting surfaces. This is a standard type of equation with many variants in physics, electrostatics being just one of them. From the solution one may calculate derived fields, and capacitance which is obtained from the total electric energy

$$E = \frac{1}{2} \int \varepsilon |\nabla \phi|^2 d\Omega \quad (7.2)$$

and the relation $E = CU^2/2$.



Figure 7.1: The geometry of the capacitor

The geometry studied is a 2D plate capacitor having the well known approximation for Capacitance $C_{app} = \varepsilon_r \varepsilon_0 A/d$. With the help of the simulation one may evaluate the fringe capacitance resulting from the end effects of the capacitor geometry. The measurements of the capacitor are 10×1 , and the distance to ground is also 1. Defining the permittivity of vacuum to be $\varepsilon_0 = 1$ the comparison to the analytical approximation becomes trivial since then then $C_{app} = 10$.

The derived fields in the StatElecSolver are computed by averaging the fields over elements – not using the Galerkin method which would provide optimal accuracy. The fields may, however, be sufficient for visualization purposes.

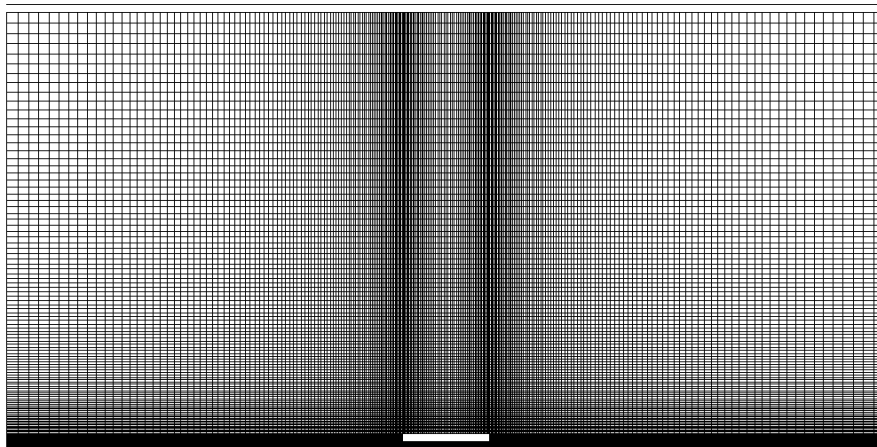


Figure 7.2: Computational mesh used in the simulation

Solution procedure

The definitions for the electrostatic equation are not loaded into ElmerGUI by default. Hence, one needs to load these before starting the simulations.

```
File
  Definitions
    Append -> electrostatics.xml
```

The additional definitions should reside in the directory `edf-extra` within the distribution. Moving the desired `xml` files to the `edf`-directory enables automatic loading of the definitions at start-up. By inspecting the definitions in the Elmer Definitions File editor one may inspect that the new definitions were really appended.

The mesh is given in ElmerGrid format in file `disc.grd`, load this file.

```
File
  Open -> disc.grd
```

You should obtain your mesh and may check that it consists of 30484 nodes and of 30348 bilinear elements.

After we have the mesh we start to go through the Model menu from the top to bottom. In the Setup we choose things related to the whole simulation such as file names, time stepping, constants etc. The steady-state simulation is carried out in 2-dimensional cartesian coordinates. For convenience we also set ϵ equal to one.

```
Model
  Setup
    Simulation Type = Steady state
    Permittivity of Vacuum = 1.0
```

In the equation section we choose the relevant equations and parameters related to their solution. In this case we'll have only the electrostatics solver.

When defining Equations and Materials it is possible to assign the to bodies immediately, or to use mouse selection to assign them later. In this case we have just one body and therefore its easier to assign the Equation and Material to it directly.

In the solver specific options we want to activate some flags that are needed to invoke the computation of derived fields. For the linear system solvers we are happy to use the defaults. One may however, try out different preconditioners (ILU1,...) or direct Umfpack solver, for example.

```
Model
  Equation
    Name = Electrostatics
    Apply to Bodies = 1
    Electrostatics
      Active = on
      Edit Solver Settings
        Solver specific options
          Calculate Electric Field = True
          Calculate Electric Energy = True
    Add
  OK
```

The Material section includes all the material parameters. In this case we only have the relative permittivity which we set to one.

```
Model
  Material
    Electrostatics
      Relative Permittivity = 1.0
    Apply to Bodies = 1
  Add
  OK
```

We have two boundary conditions for the potential at the ground and at the capacitor. For other boundaries the do-nothing boundary results to zero flux over the boundary.

```
Model
  BoundaryCondition
    Name = Ground
    Electrostatics
      Potential = 0.0
  Add
  New

  Name = Capacitor
  Electrostatics
    Potential = 1.0
  Add
```

The conditions may also be assigned to boundaries in the Boundary condition menu, or by clicking with the mouse. Here we use the latter approach as that spares us of the need to know the indexes of each boundary.

```
Model
  Set boundary properties
    Choose Ground -> set boundary condition Ground
    Choose Capacitor -> set boundary condition Capacitor
```

For the execution ElmerSolver needs the mesh files and the command file. We have now basically defined all the information for ElmerGUI to write the command file. After writing it we may also visually inspect the command file.

```
Sif
  Generate
  Edit -> look how your command file came out
```

Before we can execute the solver we should save the files in a directory. The project includes all the files needed to restart the case.

```
File
  Save Project
```

After we have successfully saved the files we may start the solver

```
Run
  Start solver
```

A convergence view automatically pops up showing relative changes of each iteration. The equation is fully linear and hence only two iterations are needed – the second one just ensures that convergence of the nonlinear level was really obtained. When the solution has finished we may start the postprocessor to view some results.

```
Run
  Start postprocessor
```

Results

From the output of the simulation one may see that the capacitance in this case was 13.70 compared to the analytical estimate of 10. Hence the fringe capacitance in this case increases the capacitance by 37 %.

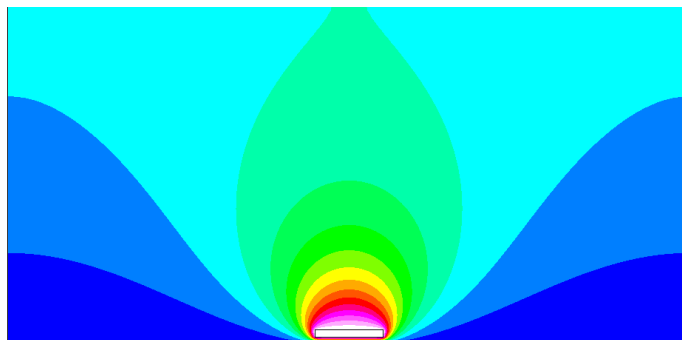


Figure 7.3: The electrostatic potential

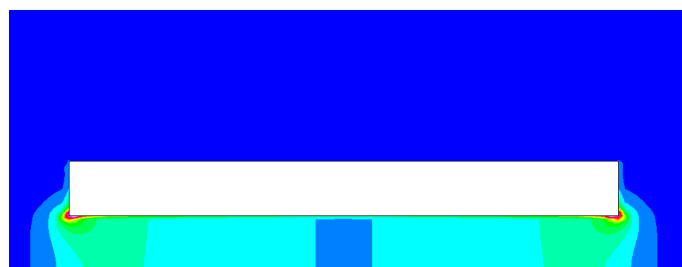


Figure 7.4: The electrostatic energy density, a close-up

Part II
non-GUI Problems

Tutorial 8

Radiation heat transfer

Directory: TemperatureRadiation
Solvers: HeatSolve
Tools: ElmerGrid, editor
Dimensions: 2D, Axi-Symmetric

Case definition

At high temperature the radiation heat transfer between walls is often the dominating heat transfer mechanism. In this tutorial we look how radiation heat transfer between cocentric cylinders is modeled.

Solution Procedure

The problem is a pure heat transfer problem that may be solved with `HeatSolve`. The view and Gebhardt factors associated with the radiation are solved as a first step in solving the equations. Thereafter the nonlinear heat equation is solved until convergence is reached.

The computational mesh is done with `ElmerGrid` in directory `radiation` with the command

```
ElmerGrid 1 2 radiation
```

The directory is given in the header of the command file

```
Header
  Mesh DB "." "radiation"
End
```

The only constant required is the Stefan-Boltzmann constant that gives the relationship between temperature and radiation power

```
Constants
  Stefan Boltzmann = 5.67e-8
End
```

The geometry is axisymmetric and the case is solved in steady state. As there is only one equation only 1 iteration for the system is required.

```
Simulation
  Coordinate System = Axi Symmetric
  Simulation Type = Steady State
  Steady State Max Iterations = 1
  Output Intervals = 1
  Output File = "radiation.result"
  Post File = "radiation.ep"
End
```

There are two bodies with the same equation but different properties.

```
Body 1
  Equation = 1
  Body Force = 1
  Material = 1
  Initial Condition = 1
End
```

```
Body 2
  Equation = 1
  Material = 2
  Initial Condition = 1
End
```

The nonlinear equation requires realistic initial conditions. Otherwise convergence may not be obtained.

```
Initial Condition 1
  Temperature = 250.0
End
```

The body force is the heating power in units W/kg.

```
Body Force 1
  Heat Source = 10000
End
```

The material properties differ only in heat conductivity. Heat capacity is not actually needed since the case is not transient.

```
Material 1
  Density = 1.0
  Heat Conductivity = 10.0
  Heat Capacity = 1.0
End
```

```
Material 2
  Density = 1.0
  Heat Conductivity = 1.0
  Heat Capacity = 1.0
End
```

The heat equation is solved with an iterative procedure that requires some relaxation for better convergence. There are two different ways to discretize the radiation. There are two keywords defining when to switch to the true Newtonian iteration which should give better convergence.

```
Solver 1
  Equation = Heat Equation
  Stabilize = True
  Linear System Solver = Iterative
  Linear System Iterative Method = BiCGStab
  Linear System Convergence Tolerance = 1.0e-12
  Linear System Max Iterations = 500
  Linear System Preconditioning = ILU
  Nonlinear System Newton After Iterations = 1
  Nonlinear System Newton After Tolerance = 1.0e-4
  Nonlinear System Max Iterations = 50
  NonLinear System Convergence Tolerance = 1.0e-8
```

```
Steady State Convergence Tolerance = 1.0e-8
Nonlinear System Relaxation Factor = 0.7
End
```

The only solver is the heat equation.

```
Equation 1
Active Solvers = 1
End
```

The radiation boundary conditions are set for two different boundaries. The first one is for the internal object and the second one for the insulation. The normal direction of the surfaces is important since a wrong direction may result to badly set problem for the view factor computation. Internal and external surfaces are very different. The normal direction may be switched with the keyword `Radiation Target Body`. A good sign of properly set case is that the view factors add up to about one.

```
Boundary Condition 1
Target Boundaries = 1
Heat Flux BC = True
Radiation = Diffuse Gray
Radiation Target Body = -1
Emissivity = 0.6
End
```

```
Boundary Condition 2
Target Boundaries = 2
Heat Flux BC = True
Radiation = Diffuse Gray
Radiation Target Body = -1
Emissivity = 0.1
End
```

The third boundary condition is the Dirichlet condition for the extranal boundary. Dirichlet conditions boost up the convergence even though the heat equation is basically well defined also with external radiation conditions.

```
Boundary Condition 3
Target Boundaries = 3
Temperature = 100.0
End
```

Results

With the given computational mesh the problem is solved in around 30 seconds. With 1 231 second order 9-noded rectangular elements the maximum temperature is 565.7 K. The corresponding results are shown in Fig. 8.1.



Figure 8.1: Temperature distribution in the radiation heat transfer problem

Tutorial 9

Eigenvalue analysis of an elastic beam

Directory: ElasticEigenValues

Solvers: StressSolve, EigenSolve

Tools: ElmerGrid, Editor

Dimensions: 3D, Steady-state

Case definition

A homogenous, elastic silicon beam of dimensions 1 m length, 0.1 m height and 0.2 m width is supported on its both ends (boundaries 1 and 2). A beam has the density 2330 kg/m^3 , Poisson ratio 0.3 and Young's modulus 10^{11} N/m^2 . The problem is to calculate the eigenvalues of the beam. Mathematically the equation to be solved is

$$-\rho\omega^2\phi = \nabla \cdot \tau(\phi)$$

where ρ is the density, ω^2 is the eigenvalue, ω is the angular frequency, ϕ is the corresponding vibration mode and τ is the stress tensor.

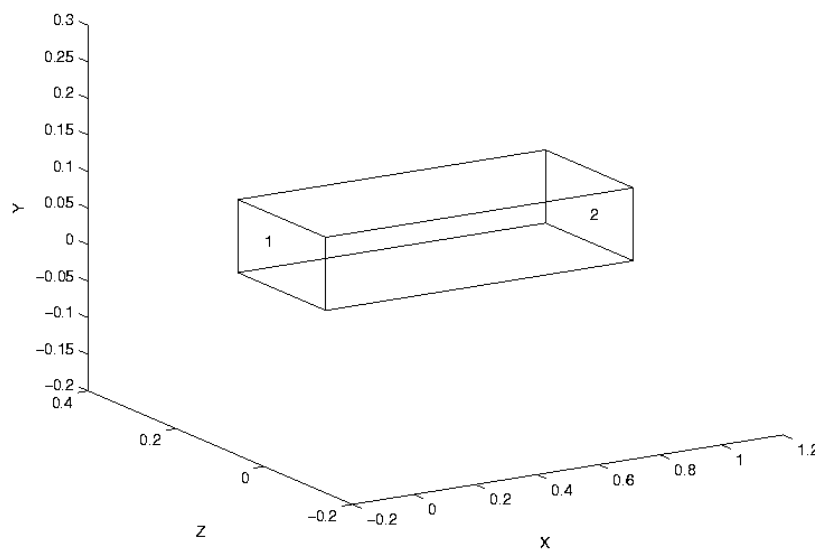


Figure 9.1: Beam.

Solution procedure

The mesh has been created by using Gambit software and it consists of 2500 elements. The mesh can be converted to Elmer format with ElmerGrid with the command

```
ElmerGrid 7 2 mesh.FDNEUT
```

This command creates the directory which contains the Elmer mesh files.

```
Header
  Mesh DB "." "mesh"
  Include Path ""
  Results Directory ""
End
```

A steady-state three-dimensional analysis is defined in the simulation section.

```
Simulation
  Coordinate System = "Cartesian 3D"
  Coordinate Mapping(3) = 1 2 3
  Simulation Type = "Steady State"
  Steady State Max Iterations = 1
  Solver Input File = "eigen_values.sif"
  Output File = "eigen_values.dat"
  Post File = "eigen_values.ep"
End
```

The geometry of the problem is simple and it includes only one body and material.

```
Body 1
  Equation = 1
  Material = 1
End
```

```
Material 1
  Youngs Modulus = 100e9
  Poisson Ratio = 0.3
  Density = 2330
End
```

The problem is solved according to linear elastic theory and due to that stress analysis is set to true.

```
Equation 1
  Stress Analysis = True
End
```

In the solver section Stress Analysis is selected. In addition, the value of the keyword Eigen Analysis have to set to true. The keyword Eigen System Values defines the number of the computed eigenvalues. The problem also is possible to solve with iterative solver but we have used direct solver in this time.

```
Solver 1
  Equation = "Stress Analysis"
  Eigen Analysis = Logical True
  Eigen System Values = Integer 5
  Linear System Solver = "direct"
```

```

Variable = "Displacement"
Variable Dofs = 3
Linear System Iterative Method = "BiCGStab"
Linear System Max Iterations = 1000
Linear System Convergence Tolerance = 1.0e-08
Linear System Abort Not Converged = True
Linear System Preconditioning = "ILU0"
Linear System Residual Output = 1
Steady State Convergence Tolerance = 1.0e-05
Nonlinear System Convergence Tolerance = 1.0e-05
Nonlinear System Max Iterations = 1
Nonlinear System Newton After Iterations = 3
Nonlinear System Newton After Tolerance = 1.0e-02
Nonlinear System Relaxation Factor = 1
Linear System Precondition Recompute = 1
End

```

The beam is supported on its both ends and therefore displacements is set to zero in all the directions.

```

Boundary Condition 1
  Target Boundaries(1) = 1
  Displacement 1 = 0
  Displacement 2 = 0
  Displacement 3 = 0
End

```

```

Boundary Condition 2
  Target Boundaries(1) = 2
  Displacement 1 = 0
  Displacement 2 = 0
  Displacement 3 = 0
End

```

After that, the problem is ready to solve.

An anisotropic model

The same problem can also be solved as an anisotropic problem which causes a couple of changes in the sif-file. First, it is reasonable to rename the files in the simulation section

```

Solver Input File = "eigen_values_aniso.sif"
Output File = "eigen_values_aniso.dat"
Post File = "eigen_values_aniso.ep"

```

For anisotropic material Young's modulus have to redefine as a matrix. In this case the matrix is defined as follows

```

Youngs Modulus
Size 6 6
  Real  200e9  60e9  60e9  0  0  0
        60e9  200e9  200e9  0  0  0
        60e9  60e9  200e9  0  0  0
        0  0  0  80e9  0  0
        0  0  0  0  80e9  0
        0  0  0  0  0  80e9
End

```

No more changes are needed in the sif-file.

Results

Both the eigenvalues of the isotropic and the eigenvalues of the anisotropic model are shown below in Elmer outputs. Figure 9.2 presents the computed eigenvectors of the beam with the isotropic model. The formula $\omega = 2\pi f$ have been used in calculating frequencies (f) (Table 9.1). According to the results the anisotropic model yielded greater eigenvalues with these values of Young's modulus.

```
EigenSolve: Computed Eigen Values:
EigenSolve: -----
EigenSolve:      1      (16737546.4275755, 0.000000000000000D+000)
EigenSolve:      2      (48175589.4544061, 0.000000000000000D+000)
EigenSolve:      3      (99674749.0526558, 0.000000000000000D+000)
EigenSolve:      4      (110392974.959463, 0.000000000000000D+000)
EigenSolve:      5      (253947166.278411, 0.000000000000000D+000)
```

Isotropic model.

```
EigenSolve: Computed Eigen Values:
EigenSolve: -----
EigenSolve:      1      (29608629.8775828, 0.000000000000000D+000)
EigenSolve:      2      (88782964.0905879, 0.000000000000000D+000)
EigenSolve:      3      (198583949.415515, 0.000000000000000D+000)
EigenSolve:      4      (205085884.544046, 0.000000000000000D+000)
EigenSolve:      5      (480903841.387323, 0.000000000000000D+000)
```

Anisotropic model.

Table 9.1: Computed frequencies.

step	isotropic	anisotropic
1	651.127 Hz	866.023 Hz
2	1104.673 Hz	1499.633 Hz
3	1588.959 Hz	2242.809 Hz
4	1672.210 Hz	2279.229 Hz
5	2536.249 Hz	3490.191 Hz

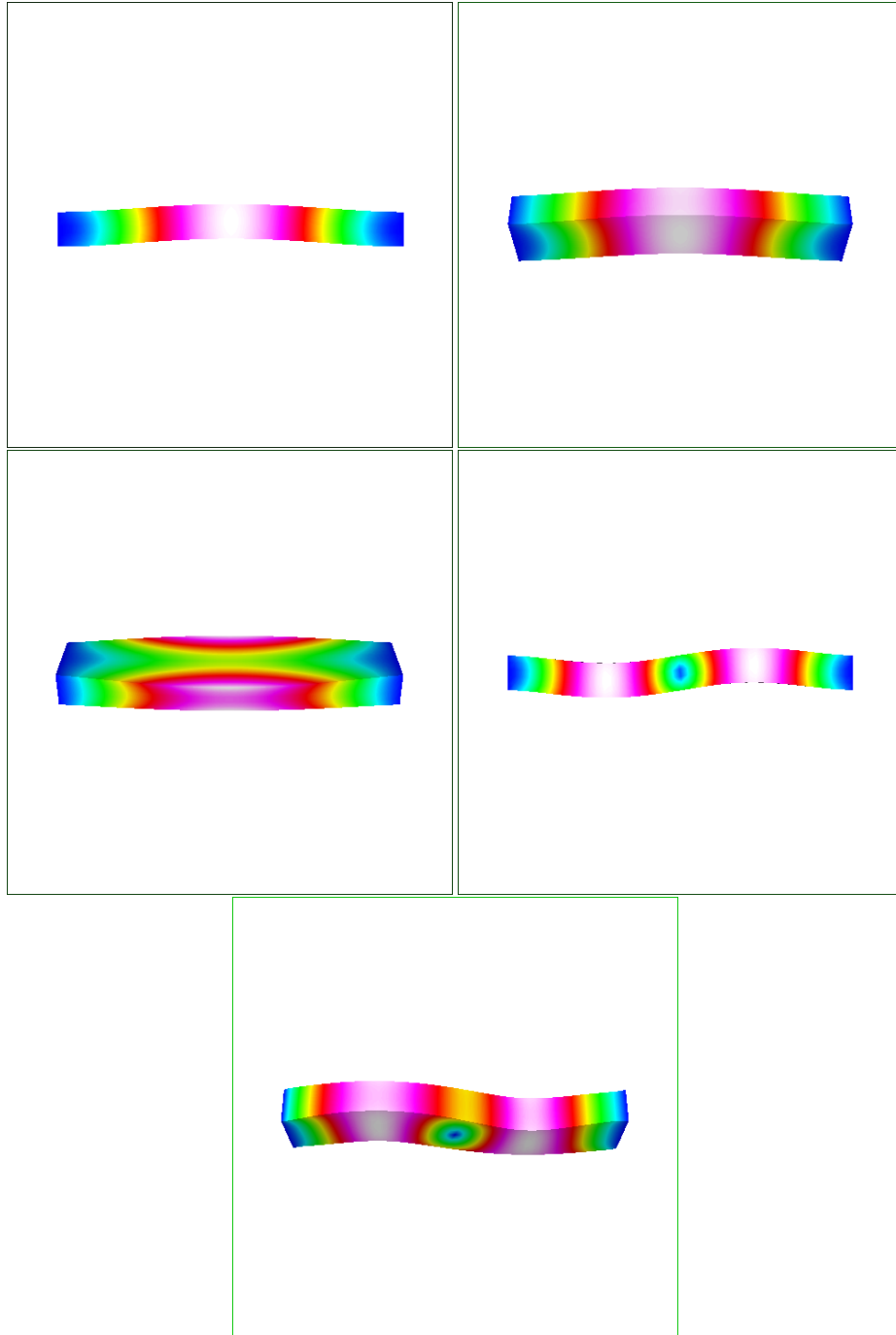


Figure 9.2: Eigenvectors

Tutorial 10

Elastic linear plate

Directory: ElasticPlateLinear
Solvers: SmitcSolver
Tools: ElmerGrid, editor
Dimensions: 2D

Case definition

This tutorial demonstrates how to use the Smitc solver to solve small deflections of plates. The Smitc solver is for elastic linear plates and uses the theory of Reissner and Mindlin.

The case under investigation is a L-shaped steel plate under pressure. The plate is shown in figure 10.1. The longer sides have the length of 2 m and the shorter 1 m . So the area of the plate is 3 m^2 . The plate has a thickness of 1 cm . We assume that on the plate there is about 15300 kg of sand. The sand is uniformly distributed on the plate and the sand stays uniformly distributed even if the plate undergoes small deflection. The sand exerts to the plate a pressure of 50000 Pa . The plate is clamped from all sides meaning that both deflection and rotation are zero on all edges.

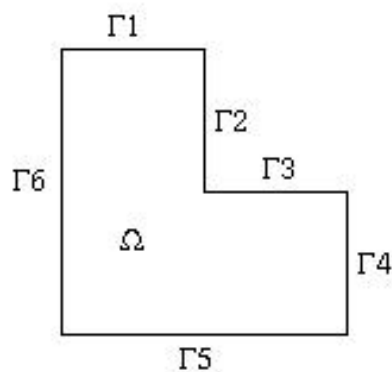


Figure 10.1: The geometry of plate and the numbering of edges.

Solution Procedure

The first thing to do is create a mesh with ElmerGrid. The definition of mesh is in the file `simple_plate.grd`. The mesh is about uniform and consist of 1000 linear square elements. The mesh is created with command

```
ElmerGrid 1 2 simple_plate
```

One thousand element should easily do the trick in this case but if more elements is needed you can edit the file `simple_plate.grd`. More specifically the line

```
Surface Elements = 1000
```

The solver input file `simple_plate.sif` starts with turning on the warnings and the definition of the proper mesh directory.

```
check keywords warn
```

```
Header
```

```
  Mesh DB "." "simple_plate"
```

```
End
```

The simulation uses 2D cartesian geometry. The simulation is not time dependent i.e. Steady State. There is no coupled solvers so only one iteration is needed. The output interval is one meaning all intervals (now there is only one interval). Numerical results are written to file `simple_plate.result` and ElmerPost file is `simple_plate.ep`.

```
Simulation
```

```
  Coordinate System = Cartesian 2D
```

```
  Simulation Type = Steady State
```

```
  Steady State Max Iterations = 1
```

```
  Output Intervals = 1
```

```
  Output File = "simple_plate.result"
```

```
  Post File = "simple_plate.ep"
```

```
End
```

There is just one body, the plate, and it uses Equation and Body Force 1 and is of Material 1.

```
Body 1
```

```
  Equation = 1
```

```
  Body Force = 1
```

```
  Material = 1
```

```
End
```

The equation block is now more than easy. It only states that we use Solver 1 to solve the equation.

```
Equation 1
```

```
  Active Solvers(1) = 1
```

```
End
```

In Body Force block we give the equations right hand side. It is the sands pressure and it is the same constant in every point.

```
Body Force 1
```

```
  Pressure = 5.0e4
```

```
End
```

In Material block we define the plates properties i.e. Poisson ratio, Young's modulus and density. We also give the plates thickness and possible pretension. Now there is no pretension.

```
Material 1
```

```
  Poisson ratio = 0.3
```

```
  Youngs modulus = 209e9
```

```
  Density = 7800.0
```

```
  Thickness = 1.0e-2
```

```
  Tension = 0.0
```

```
End
```

Next the Solver block.

- First we define that we use `SmitcSolver` and give the name of the subroutine file `Smitc` and subroutine name `SmitcSolver`.
- We name the variable `Deflection` and state that it has 3 degrees of freedom. First degree is the deflection and the remaining two are actually the components of rotation vector.
- We don't need eigen analysis nor is there any holes in the plate.
- We solve the matrix equation iteratively with stabilized biconjugate gradient method. We precondition the iteration with incomplete LU-factorization.
- Tolerance for the matrix system is $1 \cdot 10^{-8}$ and the tolerance should be achieved in less than 300 iteration.

```
Solver 1
Equation = "SmitcSolver"
Procedure = "Smitc" "SmitcSolver"

Variable = Deflection
Variable DOFs = 3

Eigen Analysis = False
Hole Correction = False

Linear System Solver = Iterative
Linear System Iterative Method = BiCGStab
Linear System Preconditioning = ILU2
Linear System Convergence Tolerance = 1.0e-8
Linear System Max Iterations = 300
End
```

Finally we give the boundary conditions. The plate has 6 edges and the edge numbering is in figure 10.1. All the edges are clamped i.e. no deflection (Deflection 1) and no rotation (Deflection 2 and 3).

```
Boundary Condition 1
Target Boundaries(6) = 1 2 3 4 5 6
Deflection 1 = 0.0
Deflection 2 = 0.0
Deflection 3 = 0.0
End
```

Results

The problem is solved in few seconds and the results are viewed with `ElmerPost`. It is possible to make `ElmerPost` to show deflection in 3D. First we determine the number of nodes. Give commands

```
math tmp = size(Deflection.1)
math n = tmp(1)
```

to `ElmerPost`. Next we put the values of deflection to nodal z-values. Deflection is rather small so the values are scaled by 50.

```
math nodes(2,0:n-1) = 50*Deflection.1
```

Result is shown in figure 10.2.

`Deflection.2` and `Deflection.3` are the x- and y-components of rotation vector. Values are transformed to vector `Rotation` with commands

```

math Rotation = 0
math Rotation(0,0:n-1) = Deflection.2
math Rotation(1,0:n-1) = Deflection.3
math Rotation(2,0:n-1) = Deflection.2*0

```

The length of vector is calculated with

```

math AbsRotation = sqrt( vdot(Rotation,Rotation) )

```

Result is shown in figure 10.2.

It is rather cumbersome to write all the commands every time you solve the problem. It is possible to write the commands to file. The file, let us name it Draw, would be

```

math tmp = size(Deflection.1);
math n = tmp(1);

math nodes(2,0:n-1) = 50*Deflection.1;

math Rotation=0;
math Rotation(0,0:n-1) = Deflection.2;
math Rotation(1,0:n-1) = Deflection.3;
math Rotation(2,0:n-1) = Deflection.2*0;

math AbsRotation = sqrt( vdot(Rotation,Rotation) );

display;

```

The file is executed in ElmerPost with command source Draw.

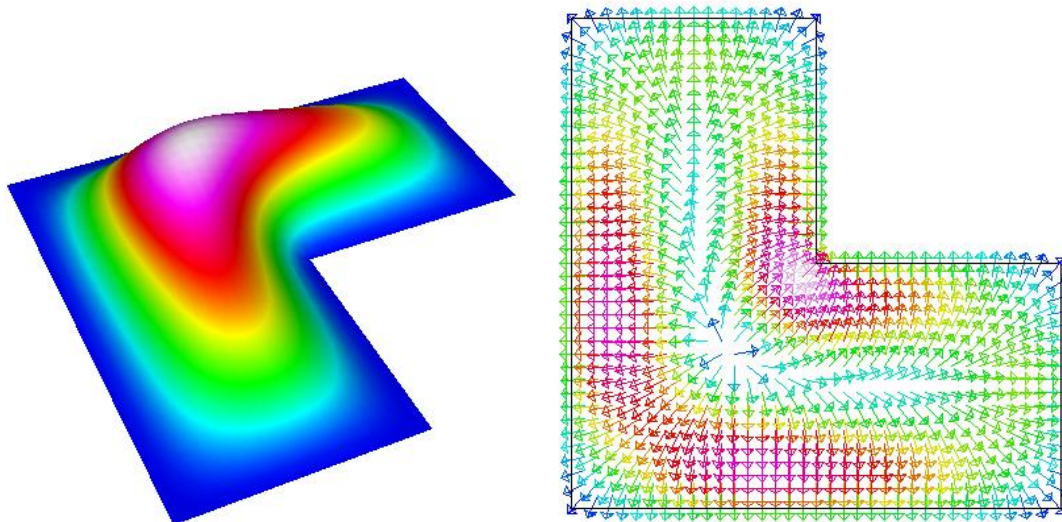


Figure 10.2: The deflection of the plate and the corresponding rotation.

Tutorial 11

Compressible flow passing a step

Directory: FlowStepCompressible
Solvers: FlowSolve, HeatSolve
Tools: ElmerGrid, Editor
Dimensions: 2D, Steady-state

Case definition

This tutorial demonstrates how to simulate the compressible air flow passing a step. The whole step has length of 1.4 m and the height of 0.2 m and the first part of it has length of 0.4 m and the height of 0.1 m (Figure 11.1). The needed material parameters of air are shown in Table 11.1. The model has three sets of boundary conditions. The air flows into the step from the inlet region and withdraws from the outlet region. The other edges of the step compose the third boundary. The flowing air is considered as an ideal gas in this case, and its density ρ depends on the pressure p and temperature T through the equation of state

$$\rho = \frac{p}{RT},$$

where R is the gas constant.

Table 11.1: Material parameters.

parameter	value
viscosity	16.7e-6 Ns/m ²
heat conductivity	0.026 W/(m·K)
heat capacity	1.01e3 J/(kg·K)
specific heat ratio	1.4
reference pressure	1e5 Pa

Solution procedure

The mesh consists of 500 rectangular elements and it is constructed using ElmerGrid with the following command

```
ElmerGrid 1 2 mesh.grd
```

This command creates the subdirectory `mesh` which contains the Elmer mesh files.

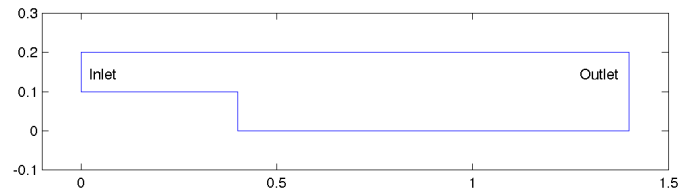


Figure 11.1: Step.

```
Header
  Mesh DB "." "mesh"
  Include Path ""
  Results Directory ""
End
```

The simulation uses 2D cartesian geometry and the problem is solved in steady state using no more than twenty steady state iterations.

```
Simulation
  Coordinate System = Cartesian 2D
  Coordinate Mapping(3) = 1 2 3
  Simulation Type = Steady
  Steady State Max Iterations = 20
  Solver Input File = "compress_step.sif"
  Post File = "compress_step.ep"
  Output File = "compress_step.dat"
End
```

The solvers are coupled and therefore the convection is computed.

```
Equation 1
  Navier-Stokes = True
  Heat Equation = True
  Convection = "Computed"
End
```

Due to the simplicity of the model only one body is needed.

```
Body 1
  Equation = 1
  Material = 1
  Initial Condition = 1
End
```

Our intention is to model compressible flow and that is why we have to set the value "Perfect Gas" for the keyword `Compressibility Model`. Furthermore, because perfect gas model has been chosen the settings `Reference Pressure` and `Specific Heat Ratio` must also be given. The Navier-Stokes equation also needs the value of viscosity and the heat equation needs the values of heat capacity and heat conductivity.

```
Material 1
  Compressibility Model = String "Perfect Gas"
  Reference Pressure = 1e5
  Specific Heat Ratio = 1.4
  Viscosity = 16.7e-6
  Heat Conductivity = 0.026
  Heat Capacity = 1.01e3
End
```

For the initial value of temperature we have chosen 300 K.

```
Initial Condition 1
  Temperature = 300
End
```

The Navier-Stokes equation is solved first. Here we give the linear system solver and convergence criteria for linear, nonlinear and steady state solution of the Navier-stokes equation. Note that we are solving for the compressible Navier-stokes equation and that is why a bubble function formulation is used for stabilization of the equation.

```
Solver 1
  Equation = "Navier-Stokes"
  Linear System Solver = "Iterative"
  Linear System Iterative Method = "BiCGStab"
  Linear System Max Iterations = 500
  Linear System Convergence Tolerance = 1.0e-08
  Linear System Abort Not Converged = True
  Linear System Preconditioning = "ILU2"
  Linear System Residual Output = 1
  Steady State Convergence Tolerance = 1.0e-05
  Bubbles = Logical True
  Nonlinear System Convergence Tolerance = 1.0e-05
  Nonlinear System Max Iterations = 1
  Nonlinear System Newton After Iterations = 3
  Nonlinear System Newton After Tolerance = 1.0e-02
  Nonlinear System Relaxation Factor = 1
End
```

The corresponding parameters for the solver of the heat equation are defined in the following solver section.

```
Solver 2
  Equation = "Heat Equation"
  Variable = "Temperature"
  Linear System Solver = "Iterative"
  Linear System Iterative Method = "BiCGStab"
  Linear System Max Iterations = 350
  Linear System Convergence Tolerance = 1.0e-08
  Linear System Preconditioning = "ILU0"
  Linear System Residual Output = 1
  Steady State Convergence Tolerance = 1.0e-05
```



```

Bubbles = Logical True
Nonlinear System Convergence Tolerance = 1.0e-05
Nonlinear System Max Iterations = 1
Nonlinear System Newton After Iterations = 3
Nonlinear System Newton After Tolerance = 1.0e-02
Nonlinear System Relaxation Factor = 1
End

```

Finally, the boundary conditions are specified. There are three sets of boundary conditions, so three Boundary Condition sections are needed. The first one is used to prescribe the boundary conditions in the inlet region. Note that we have defined the x-velocity and temperature as a variable of y-coordinate. This is done by setting different values for the x-velocity and temperature (the numerical values of the second column between the words Real and End) in the different y-points (the numerical values of the first column between words Real and End) of the inlet region. This kind of procedure prevents occurring singularities in the corner points of the inlet region. In addition, this kind of definition is more realistic than a condition, in which the values of the x-velocity and temperature remain the same in the whole inlet region.

```

Boundary Condition 1
  Target Boundaries = 1
  Velocity 1 = Variable Coordinate 2
    Real
      0.1    0
      0.15  0.02
      0.2    0
    End
  Velocity 2 = 0
  Temperature = Variable Coordinate 2
    Real
      0.1    300
      0.15  350
      0.2    300
    End
End

```

After the rest boundary conditions have been defined the problem is ready to solve.

```

Boundary Condition 2
  Target Boundaries = 2
  Velocity 2 = 0
End

```

```

Boundary Condition 3
  Target Boundaries = 3
  Velocity 1 = 0
  Velocity 2 = 0
  Temperature = 300
End

```

Results

Figure 11.2 presents the temperature distribution of the step in steady state. The maximum and minimum values of x- and y-velocities are also given as a result and they are shown in Table 11.2.

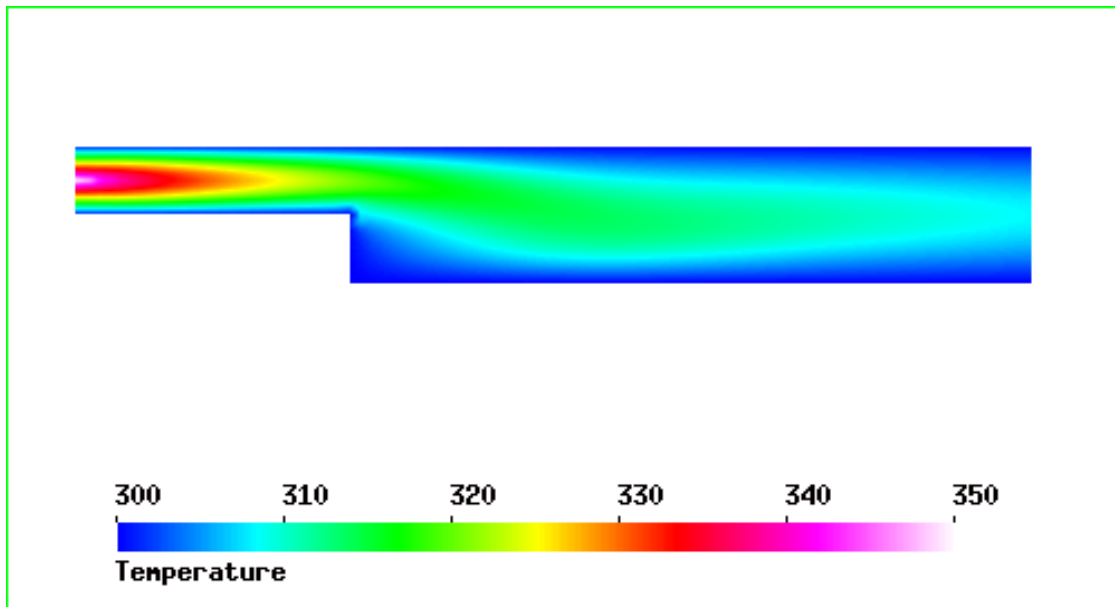


Figure 11.2: Step.

Table 11.2: Computed velocities.

velocity	value
min x-velocity	-0.0014 m/s
min y-velocity	-0.0016 m/s
max y-velocity	0.0008 m/s

Tutorial 12

Flow through a hole – determining the acoustic impedance

Directory: FlowResistance

Solvers: FlowSolve

Tools: ElmerGrid, editor

Dimensions: 3D, Steady-state

Case definition

The problem at hand consists of finding the resistance that a fluid faces as it is forced to flow through a hole. The flow resistance is stated by the ratio of pressure drop over the hole and the input velocity. In microsystem modeling, the hole resistance is often needed to analyse the gas damping effect in perforated structures. Here, the contribution of the holes is homogenised over the perforated structure based on a single hole resistance. For homogenisation in Elmer, the specific acoustic impedance is used to represent the flow resistance. Specific acoustic impedance z_h is defined as

$$z_h = \frac{p}{v} = \frac{F}{vA_h}, \quad (12.1)$$

where F is the net force due to gas on the moving surface, v is the velocity of the gas on the same surface, and A_h is the area of the moving surface. The calculation is best performed in a unit cell of the geometry.

In order to the homogenisation to be possible, the dependence of input velocity and the net force should be linear. Further, there should not be a phase change between these two quantities. These conditions are satisfied when the flow is incompressible. In a linear case, the fluid flow can be treated with the linear form of Navier-Stokes equations called the Stokes equation

$$\rho \frac{\partial \vec{u}}{\partial t} - \nabla \cdot (2\eta \bar{\varepsilon}) + \nabla p = \rho \vec{f}, \quad (12.2)$$

where \vec{u} is the unknown velocity field, p is the pressure, η is the viscosity of the fluid, $\rho \vec{f}$ is a body force and $\bar{\varepsilon}$ is the linearised strain tensor. Note, that the stationary version of the above equation can be used in homogenisation calculations.

The condition for Stokes equation to apply is that the Reynolds number Re of the problem should be small

$$Re = \frac{\rho UL}{\eta}, \quad (12.3)$$

where ρ is density of the fluid and U and L are, respectively, the velocity and length scales of the problem.

The issue of compressibility is more difficult to answer. A classical condition for the compressibility is that the Mach number Ma of the problem should be small

$$Ma = \frac{U}{a} < 0.3, \quad (12.4)$$

where a is the speed of sound in the gas in operating conditions and the value 0.3 is often stated limit for a small Mach number (actually, the condition is that Ma^2 has to be small). Also the frequency and amplitude of the oscillations of the system have an effect on the validity of the linearity and incompressibility assumptions, since they affect the velocity scale of the problem.

However, also other factors have an effect on the compressibility of the gas. In microsystems, the viscous effects on pressure, or even temperature changes, can affect the density of the gas. A condition for viscous pressure changes is that Ma^2/Re has to be small, and for temperature, in addition, that the Prandtl number Pr may not be too large

$$Pr = \frac{\eta c_p}{k}, \quad (12.5)$$

where c_p is the heat capacity (*ie.* specific heat) in constant pressure and k is the thermal conductivity.

The conditions listed here for the flow to be approximately incompressible are only an overview and the validity of incompressibility assumption should be considered in each case separately. In microsystems, refer for example to the article M. Gad-el-Hak, *J. Fluids Eng.*, 121, 5–33, 1999. Additionally, it is advisable to perform numerical checks on the issue.

One final point on the applicability of the Stokes (or Navier-Stokes) equations is the effect of gas rarefaction. If the dimensions of the problem are very small the continuity assumption may not be valid anymore. The importance of the gas rarefaction effects are given by the Knudsen number Kn

$$Kn = \frac{\mathcal{L}}{\bar{L}}, \quad (12.6)$$

where \mathcal{L} is the mean free path of the gas molecules. The mean free path depends inversely on ambient pressure, which has to take into account in stating the Knudsen number. For Knudsen numbers close to and less than 1, slip boundary conditions should be used.

To summarise, the motivation of this tutorial is to perform a linear incompressible simulation of fluid flowing through a hole. The wake for the flow is a constant velocity boundary condition for a boundary before the hole. On the same boundary, the force caused by the fluid is recorded. These two quantities can then be used to determine the specific acoustic impedance of a single hole. The constant velocity boundary condition may be interpreted as describing a moving wall with small displacement. In this particular tutorial, a symmetrical quadrant of a square-shaped hole is used.

Solution procedure

The solution for the problem is found by solving iteratively the Stokes equation. Nonlinear iterations are not needed, since the problem is linear.

The computational mesh should include enough free space after the hole so that any artificial effects due to the boundaries of the mesh are avoided. In this tutorial, the geometry is created and meshed using the ElmerGrid program by the command `elmergrid 1 2 hole.grd`. The default mesh consists of about 12000 nodes and 10500 eight-noded hexahedrons.

The header section of solver input file includes only the location of the mesh files.

```
Header
  Mesh DB "." "hole"
End
```

In the simulation section, a steady-state three-dimensional analysis is defined.

```
Simulation
  Coordinate System = Cartesian 3D
  Simulation Type = Steady State
  Steady State Max Iterations = 1
  Output File = "flow.result"
  Post File = "flow.ep"
End
```

The geometry contains only one body and no body forces or initial conditions are present. The body section reads thus as follows.

```

Body 1
  Equation = 1
  Material = 1
End

```

For solving the flow patterns the Navier-Stokes solver is used but the nonlinearity through convection is switched off in the equation block. Also, solvers for the fluidic force and saving data are enabled.

```

Equation 1
  Active Solvers(3) = Integer 1 2 3
  NS Convect = False
End

```

Just a single iteration of the Navier-Stokes solver is needed, since the equation is linear. This can be verified by switching the number of nonlinear iterations to a value more than one, and observing the change in solution between iteration steps.

```

Solver 1
  Equation = Navier-Stokes
  Variable = Flow Solution
  Variable DOFs = 3
  Linear System Solver = Iterative
  Linear System Iterative Method = BiCGStab
  Linear System Preconditioning = ILU0
  Linear System Max Iterations = 200
  Linear System Convergence Tolerance = 1.0e-08
  Stabilize = True
  Nonlinear System Convergence Tolerance = 1.0e-05
  Nonlinear System Max Iterations = 1
  Nonlinear System Newton After Iterations = 3
  Nonlinear System Newton After Tolerance = 1.0e-08
  Nonlinear System Relaxation Factor = 1.0
  Steady State Convergence Tolerance = 1.0e-05
End

```

The fluidic force solver needs to be run only once, after the flow solution is finished. With the keyword `Calculate Viscous Force` it is possible to define whether the viscous forces of the fluid are included in the force or not. If this is set to false, only the pressure integral is calculated.

```

Solver 2
  Exec Solver = After All
  Equation = Fluidic Force
  Procedure = "FluidicForce" "ForceCompute"
  Calculate Viscous Force = True
End

```

The final solver is used to save data from the analysis. With the following definitions, the input velocity and the net force on the input boundary as well as the area of the boundary are written into a file called `flowdata.dat`.

```

Solver 3
  Exec Solver = After All
  Equation = SaveScalars
  Procedure = "SaveData" "SaveScalars"
  Filename = "flowdata.dat"
  Save Variable 1 = Velocity 3
  Save Coordinates(1,2) = 0.0 0.0
End

```

The fluid is defined to be air. Note the Elmer MEMS units used.

```
Material 1
  Name = Air
  Density = 1.293e-12
  Viscosity = 1.67e-5
End
```

Finally, the boundary conditions. BC 1 defines the input boundary, where also the fluidic force is calculated. BCs 2 and 4 define the symmetry boundaries, BC 3 defines the no-slip conditions for the walls, and BC 5 defines an open boundary.

```
Boundary Condition 1
  Target Boundaries = 4
  Velocity 1 = 0.0
  Velocity 2 = 0.0
  Velocity 3 = 1.0e3
  Calculate Fluidic Force = True
End
```

```
Boundary Condition 2
  Target Boundaries(2) = 8 10
  Velocity 2 = 0.0
End
```

```
Boundary Condition 3
  Target Boundaries(4) = 1 2 3 7
  Velocity 1 = 0.0
  Velocity 2 = 0.0
  Velocity 3 = 0.0
End
```

```
Boundary Condition 4
  Target Boundaries(2) = 6 9
  Velocity 1 = 0.0
End
```

```
Boundary Condition 5
  Target Boundaries = 5
  Pressure = 0.0
End
```

Slip boundary conditions

The same simulation can also be performed using slip boundary conditions. These are appropriate, as stated in introduction, when the Knudsen number is between 10^{-3} and 1. The slip boundary condition implemented in Elmer is of first order

$$S \cdot \vec{u} = \bar{\bar{\sigma}} \cdot \vec{n}, \quad (12.7)$$

where S is a vector containing the slip coefficients s_i for each velocity component, μ is the viscosity, and $\bar{\bar{\sigma}}$ is the stress tensor. For Newtonian fluids and for tangential directions of the boundary this gives

$$s_i u_i = \mu \frac{\partial u_i}{\partial n}, \quad (12.8)$$

where s_i and u_i refer to the same tangential component of the slip coefficient and the flow velocity.

The value of the slip coefficient is related to the mean free path of the gas molecules λ . For example, Maxwell's first order slip boundary condition may be used (as in *e.g.* A. Beskok, *Num. Heat Transfer*, B, 40, 451–471, 2001):

$$u_i = \frac{2 - \sigma_v}{\sigma_v} \lambda \frac{\partial u_i}{\partial n}, \quad (12.9)$$

where σ_v is the tangential momentum accommodation coefficient, which models the momentum exchange of gas molecules and the surface. The accommodation coefficient is dependent on the gas and on the surface, and recent measurements give a result of $\sigma_v \simeq 0.80$ for various monoatomic gases such as Argon in contact with prime Silicon crystal.

The slip coefficient of Elmer can finally be written as

$$s_i = \frac{\mu}{\lambda} \frac{\sigma_v}{2 - \sigma_v}. \quad (12.10)$$

The mean free path is defined as

$$\lambda = \frac{\mu}{\rho} \sqrt{\frac{\pi M}{2RT}}, \quad (12.11)$$

where ρ is density, M is the molar mass, T is the temperature, and $R = 8.3145 \text{ J/mol K}$ is the molar gas constant.

In the Elmer analysis, only a few changes in the `sif`-file are needed to make the slip conditions active. The flow force boundary conditions have to be turned on and the numerical value of the slip coefficient has to be defined on each boundary (here $s = 2\text{e-}4$ is used for air). Further below is a list of the Boundary Condition blocks. Note that there are more BCs than in the no-slip simulation, since a separate condition is needed for surfaces oriented differently in space.

Generally, a normal-tangential orientation scheme for the boundary conditions are needed, since the surfaces are not necessarily having a normal vector pointing in one of the coordinate directions. This would be done for each such boundary by the line

```
Normal-Tangential Velocity = True
```

after which the Velocity component 1 points to the normal direction and the other components to the tangential directions.

```
! Definitions for slip boundary conditions:
```

```
Boundary Condition 1
```

```
Target Boundaries = 4
```

```
Flow Force BC = True
```

```
Slip Coefficient 1 = 2e-4
```

```
Slip Coefficient 2 = 2e-4
```

```
Velocity 3 = 2.0e3
```

```
Calculate Fluidic Force = True
```

```
End
```

```
Boundary Condition 2
```

```
Target Boundaries(2) = 8 10
```

```
Velocity 2 = 0.0
```

```
End
```

```
Boundary Condition 3
```

```
Target Boundaries(2) = 2 3
```

```
Flow Force BC = True
```

```
Velocity 3 = 0.0
```

```
Slip Coefficient 1 = 2e-4
```

```
Slip Coefficient 2 = 2e-4
```

```
End
```

```

Boundary Condition 4
  Target Boundaries(2) = 6 9
  Velocity 1 = 0.0
End

Boundary Condition 5
  Target Boundaries = 5
  Pressure = 0.0
End

Boundary Condition 6
  Target Boundaries = 1
  Flow Force BC = True
  Velocity 1 = 0.0
  Slip Coefficient 2 = 2e-4
  Slip Coefficient 3 = 2e-4
End

Boundary Condition 7
  Target Boundaries = 7
  Flow Force BC = True
  Velocity 2 = 0.0
  Slip Coefficient 1 = 2e-4
  Slip Coefficient 3 = 2e-4
End

```

Results

The computation takes about 200 cpu seconds on an AlphaServer with 1 GHz central processor when trilinear elements are used. The results for two different input velocities taken from the file `flowdata.dat` are summarised in Table 12.1. Also the specific acoustic impedance z_h is calculated in the table. The results of slip and no-slip simulations are also compared. Note that for the force, only the component perpendicular to the surface should be used since the other components cancel out due to symmetry. The values in the table are again given in Elmer MEMS units.

Table 12.1: Results of flow simulations for two input velocities

v	slip model	F_z	z_h
$1.0 \cdot 10^3$	no-slip	36.13	$1.45 \cdot 10^{-3}$
$2.0 \cdot 10^3$	no-slip	72.25	$1.45 \cdot 10^{-3}$
$1.0 \cdot 10^3$	slip	29.30	$1.17 \cdot 10^{-3}$
$2.0 \cdot 10^3$	slip	58.60	$1.17 \cdot 10^{-3}$

The identical values obtained for the specific acoustic impedance in Table 12.1 prove by no means that the flow in reality is linear, since this was the assumption and the simulation performed can and should not reveal any nonlinear behavior. The results indicate, though, that allowing slip on the boundaries reduces the resistance that the fluid faces. This example shows that in microsystems, as the dimension of the smallest flow channel is in the range of a micrometer, it is reasonable to use slip boundary conditions for the velocity.

Finally, a picture of the results obtained with no-slip conditions is presented. The Fig. 12.1 shows a lot of pressure isosurfaces which are coloured using the absolute value of the velocity.

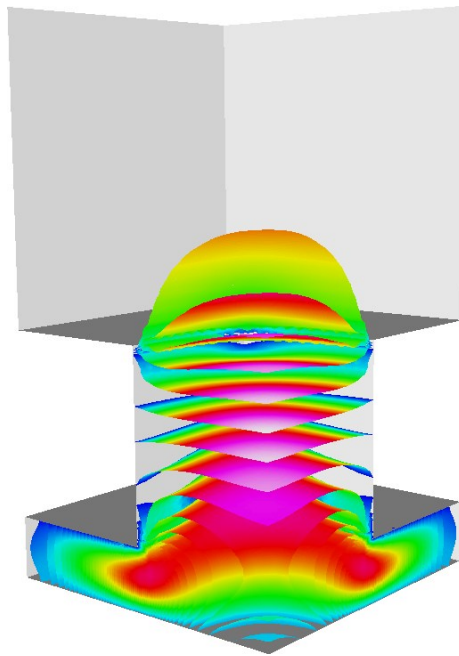


Figure 12.1: The linear flow results.

Tutorial 13

Electrostatics

Directory: Electrostatics

Solvers: StatElecSolve, ElectricForce

Tools: ElmerGrid, editor

Dimensions: 3D, Steady-state

Case definition

This case presents solving the Poisson equation for electric potential and calculating appropriate derived quantities, such as capacitance, based on the result. The geometry studied is a symmetric quadrant of a plane capacitor having a rectangular hole in another plate. A setting of this kind can be used to study the effects of geometrical features on the capacitance and on the electrostatic force, which both are meaningful quantities for coupled simulations in *e.g.* microsystems.

Solution procedure

The mesh is constructed using ElmerGrid with the following command

```
ElmerGrid 1 2 elmesh.grd
```

The mesh is extended above the hole to avoid undesired boundary effects. The geometry is presented in the Figure 13.1

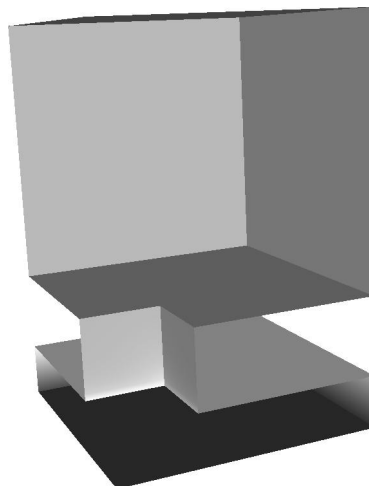


Figure 13.1: The geometry of problem.

The simulation problem includes a single body, and thus one material and one equation set, as well as three solvers. The solvers are used to compute the electric potential and related quantities, to calculate the electric force, and to save relevant data into a file. This tutorial is defined in Elmer MEMS units. The sif-file is presented below.

```
Check Keywords Warn
```

```
Header
```

```
  Mesh DB "." "elmesh"
```

```
End
```

Only a single steady state iteration is needed, since the Poisson equation is linear.

```
Simulation
```

```
  Coordinate System = Cartesian 3D
```

```
  Simulation Type = Steady State
```

```
  Steady State Max Iterations = 1
```

```
  Output File = "elstatics.result"
```

```
  Post File = "elstatics.ep"
```

```
End
```

The permittivity of vacuum has to be defined in the Constants section.

```
Constants
```

```
  Permittivity Of Vacuum = 8.8542e-12
```

```
End
```

```
Body 1
```

```
  Equation = 1
```

```
  Material = 1
```

```
End
```

Electric energy density is added into the results in Equation section. This allows energy density to be visualised in ElmerPost. Note also, that calculating electric flux (or the electric displacement field) is disabled in the Solver 1 block. Further, the potential difference used in calculating the capacitance of the system has to be defined in this section. This should be the same as the boundary conditions define for the capacitance calculation to be sensible.

```
Equation 1
```

```
  Active Solvers(2) = 1 2
```

```
  Calculate Electric Energy = True ! (default False)
```

```
End
```

```
Solver 1
```

```
  Equation = Stat Elec Solver
```

```
  Variable = Potential
```

```
  Variable DOFs = 1
```

```
  Procedure = "StatElecSolve" "StatElecSolver"
```

```
  Calculate Electric Field = True ! (default True)
```

```
  Calculate Electric Flux = False ! (default True)
```

```
  Potential Difference = 1.0e6
```

```
  Linear System Solver = Iterative
```

```
  Linear System Iterative Method = BiCGStab
```

```
  Linear System Max Iterations = 200
```

```
  Linear System Convergence Tolerance = 1.0e-07
```

```
  Linear System Preconditioning = ILU1
```

```
  Linear System ILUT Tolerance = 1.0e-03
```

```

Nonlinear System Max Iterations = 1
Nonlinear System Convergence Tolerance = 1.0e-4
Nonlinear System Newton After Tolerance = 1.0e-3
Nonlinear System Newton After Iterations = 10
Nonlinear System Relaxation Factor = 1
Steady State Convergence Tolerance = 1.0e-4
End

```

The static electric force solver does not need a lot of information:

```

Solver 2
Equation = Electric Force
Procedure = "ElectricForce" "StatElecForce"
End

```

Finally, some data is saved in file scalars.dat in working directory.

```

Solver 3
Exec Solver = After All
Equation = SaveScalars
Procedure = "SaveData" "SaveScalars"
Filename = "scalars.dat"
End

```

Only the relative permittivity of the material has to be defined.

```

Material 1
Relative Permittivity = 1
End

```

The boundary conditions include the values of electric potential (voltage) and indication on which boundary the electric force should be calculated. On all the other boundaries a natural boundary condition is used, basically stating that the electric flux through these boundaries is zero.

```

Boundary Condition 1
Target Boundaries = 4
Potential = 0.0
Calculate Electric Force = True
End

```

```

Boundary Condition 2
Target Boundaries = 3
Potential = 1.0e6
End

```

Results

The results obtained for capacitance and electric force are compared to those of a complete plane capacitor. For a plane capacitor, the capacitance is

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d}, \quad (13.1)$$

and the electrostatic force is

$$F_e = \frac{1}{2} \varepsilon_r \varepsilon_0 \frac{A}{d^2} \Phi^2, \quad (13.2)$$

where ε_r is the relative permittivity, ε_0 is the permittivity of vacuum, A is the area of a capacitor plate, d is the separation of the capacitor plates, and Φ is the potential difference between the plates.

Table 13.1: Comparison of numerical results to analytic values

	simulation	analytic	ratio
Capacitance	$2.1361 \cdot 10^{-10}$	$2.2136 \cdot 10^{-10}$	0.965
Electric Force	$1.0406 \cdot 10^2$	$1.1068 \cdot 10^2$	0.940

The results of the simulation as well as the comparison to the complete plane capacitor values are shown in Table 13.1 (in Elmer MEMS units). Note that the fringe fields on capacitor edges are not calculated. This would require much larger mesh extending outside the capacitor boundaries.

Finally, a picture of the results is presented. The Figure 13.2 shows the isosurfaces of the electric potential with the color marking the strength of the electric field. From the picture it is clearly seen that the electric field is constant between the plates except for the proximity of the hole which causes weakening of the field magnitude. There are also strong electric fields at the edges of the hole.

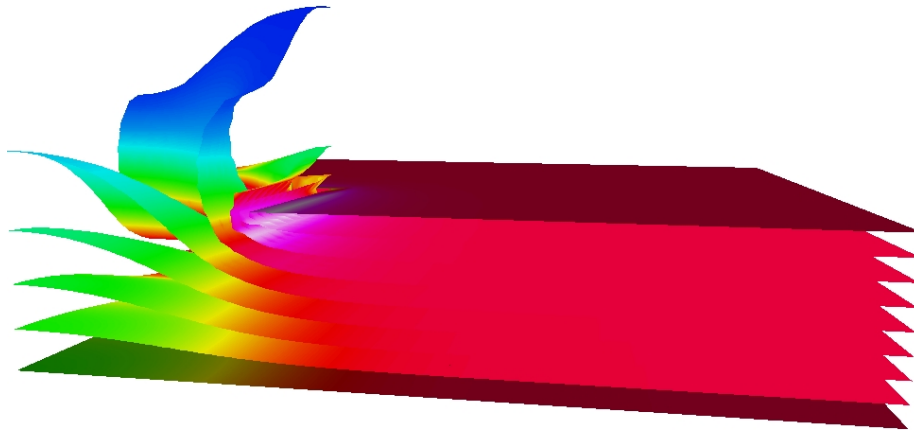


Figure 13.2: Isosurfaces of the potential coloured with electric field magnitude.

Tutorial 14

Lossless acoustic waves

Directory: AcousticWaves

Solvers: HelmholtzSolve

Tools: ElmerFront

Dimensions: 2D, Harmonic

Introduction

Elmer provides two alternative ways of conducting acoustic analyses in the frequency domain. Firstly, one may simply use the Helmholtz equation which is based on the assumption of lossless flow, i.e. the effects of viscosity and heat conduction are assumed to be negligible. More refined analyses where these effects are taken into account may be carried out by using the specific solver for the set of time-harmonic dissipative acoustic equations. The aim of this tutorial is to demonstrate the usage of the solver for the basic Helmholtz equation, which is frequently taken as the starting point in acoustic analyses.

Case description

In this problem the fluctuations of the pressure in an air-filled cavity shown in Figure 14.1 are considered. The cavity is connected with the surrounding air by an open narrow pipe. The pressure fluctuations are generated by a vibrating membrane on the boundary Γ_S with the frequency of the motion being $f = 100$ Hz. The remaining parts of the boundary are assumed to be rigid walls. In addition, the effects of gravity are assumed to be negligible.

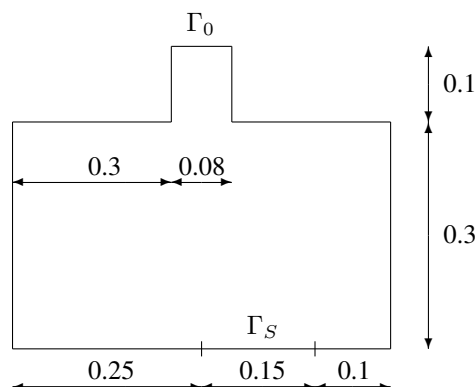


Figure 14.1: The geometry of the cavity.

Suitable boundary conditions in terms of the pressure must be given. On the rigid walls the pressure flux is prescribed to vanish which corresponds to the assumption that there is no velocity in the direction normal to the boundary. At the open end Γ_0 the impedance boundary condition suitable for forward traveling plane waves is given by setting $Z = -c$ with c being the sound speed. We assume that $c = 343$ (m/s). Finally, the wave source is given by defining a non-vanishing pressure flux on the corresponding part of the boundary. We take simply $\nabla P \cdot \vec{n} = 1$ where P is the (complex) amplitude of the pressure and \vec{n} is the outward unit normal to the boundary.

Solution procedure

- Before starting Elmer copy the geometry file (`domain.egf`) to the working directory and then launch Elmer Front by giving the command

```
ElmerFront
```

- Open the geometry file by choosing Open Cad-file in the File menu. To enable browsing with the mouse click the button on the right-hand side of the field where the file name may be written. Here the correct Cad file type is Elmer. Give also the model name (for example `helmholtz`) and write the path of the working directory in the Model directory field.
- Select the equation to be solved by selecting Equations in the Problem menu. Choose the Helmholtz equation and press Add button.
- Define the angular frequency for the simulation by selecting Simulation parameters in the Problem menu. Enter the value 628.3 to the field and accept the value by clicking OK button.
- Define the sound speed for the medium by selecting Materials in the Model menu. Enter the value 343 for the sound speed and press Add button.
- Prescribe the boundary conditions by selecting Boundary conditions in the Model menu. Select (with the mouse) Boundary1 and give the value for the boundary flux:

```
Wave flux Re = 1
Wave flux Re = 0
```

Finally, press Add button. Then proceed to give the other boundary conditions in a similar manner (the value for the pressure is prescribed).

- Create a finite element mesh by selecting Define mesh in the Mesh menu. To begin with give a name for the mesh. Define the number of element edges on each boundary and then create the mesh by pressing Generate Mesh button.
- The problem may now be solved by selecting Solver in the Run menu.
- After the solution is done, view the results by selecting the Postprocessor from the Run menu.

```
Run -> Postprocessor
```

- To save the created model, select Save model file from the File menu.

```
File -> Save model file
```

Results

Using a mesh consisting of 3900 (quadratic) elements with 7601 nodes the difference of the maximum and the minimum value of the pressure is found to be $\Delta p \approx 0.205$

Tutorial 15

Induction heating of a graphite crucible

Directory: InductionHeating

Solvers: StatMagSolve

Tools: ElmerGrid, editor

Dimensions: 2D, Axi-Symmetric

Case definition

At high temperatures the most practical method to heat up the crucible is by electromagnetic induction. The induction coil generates an alternating current that flows through the crucible. The Ohmic resistance encountered by this current dissipates energy, thereby directly heating the crucible via internal heat generation.

The tutorial case is a simple axi-symmetric crucible that could be used, for example, to grow silicon carbide (SiC) by the sublimation method. The crucible is made of dense graphite and isolated by porous graphite. At the bottom of the crucible there is some SiC powder. The physical properties of the material are given in Table 15.1. The dimensions of the induction heating crucible are given in Table 15.2. Additionally, the powder thickness is 1.0 cm and there are 10 spirals in the coil. The frequency of induction heating f is 50 kHz and the current I is 10 A. The permeability of the space is $4\pi 10^{-7}$ if the other variables are in SI-units.

Solution Procedure

At low frequencies the free charges may be neglected and the induction heating problem may be solved in terms of a magnetic vector potential. The proper solver to do this is `StatMagSolver`. However, the induction heating problem can only be modeled if the helicity of the coil is neglected and an average current density is assumed. This current density may be computed easily when the area of the coil is known $j_0 = nI/A$, where A is the coil area.

The mesh for this problem may easily be created by ElmerGrid. The provided mesh is quite sufficient for this case but for higher frequencies the mesh should be tuned to solve the thin boundary layers. The computational mesh is created from file `crucible.grd` by the command

```
ElmerGrid 1 2 crucible
```

Table 15.1: Material parameters of the crucible

material	ε	κ [W/mk]	σ (1/ Ω m)
graphite	0.7	10.0	2.0E4
insulation	0.9	1.0	2.0E3
powder	0.5	25.0	1.0E4

Table 15.2: Dimensions of the crucible

body part	r_{inner}	r_{outer}	h_{inner}	h_{outer}
graphite	2.0	2.5	6.0	8.0
insulation	2.5	4.0	8.0	12.0
coil	5.0	5.5		8.0

The mesh consists of 5 different bodies which need 4 different materials sets. Only one set of boundary conditions are required for the external boundary. Thus the header information of the command file is as follows

```
Header
  Mesh DB "." "crucible"
  Include Path ""
  Results Directory ""
End
```

In the Simulation section the coordinate system and time dependency is set, among other things. Also we know that the equation is linear and therefore only one steady state iteration is required. If the electric properties depend on the magnitude of the field several iterations are required.

```
Simulation
  Coordinate System = "Axi Symmetric"
  Simulation Type = Steady State
  Steady State Max Iterations = 1
  Output File = "crucible.result"
  Post File = "crucible.ep"
End
```

In the Constants section the permittivity of vacuum must be given.

```
Constants
  Permittivity Of Vacuum = 8.8542e-12
End
```

In the differential equation for the magnetic vector potential the source is the current density. Thus, it is given in the Body Force section.

```
Body Force 1
  Current Density = 2.5e5
End
```

In the Body section the different bodies are assigned with correct equation sets and material parameters, for example

```
Body 3
  Name = "Insulation"
  Equation = 1
  Material = 2
End
```

In the Equation block all the relevant solvers are set to active.

```
Equation
  Name = "Vector Potential Equation"
  Active Solvers = 1
End
```

The only solver in this simple tutorial is the solver for the magnetic vector potential. Look for the relevant model manual for information about the options. Here the equation is solved iteratively and the local Joule heating and magnetic flux are computed as a postprocessing step. The Joule heating is scaled so that the total heating power is 3.0 kW. This option may be used when the total heating efficiency is known. The nonlinear solver parameters are not really needed as the material parameters are constant. Sometimes the parameters may depend on the magnetic field and thus the nonlinear problem must be solved iteratively.

```
Solver 1
  Equation = Potential Solver
  Variable = Potential
  Variable DOFs = 2

  Angular Frequency = Real 50.0e3
  Calculate Joule Heating = Logical True
  Calculate Magnetic Flux = Logical True
  Desired Heating = Real 3.0e3

  Procedure = "StatMagSolve" "StatMagSolver"
  Linear System Solver = Iterative
  Linear System Iterative Method = BiCGStab
  Linear System Max Iterations = 300
  Linear System Convergence Tolerance = 1.0e-10
  Linear System Preconditioning = ILU1
  Linear System ILUT Tolerance = 1.0e-03
  Linear System Residual Output = 1
  Nonlinear System Max Iterations = 1
  Nonlinear System Convergence Tolerance = 1.0e-6
  Nonlinear System Relaxation Factor = 1
  Steady State Convergence Tolerance = 1.0e-6
End
```

In the Material sections all the necessary material parameters are given, for example

```
Material 2
  Name = "Insulation"
  Electric Conductivity = 2.0E3
End
```

The magnetic field must vanish at infinity. Unfortunately the computational domain is bounded and therefore the infinite distance becomes very finite. A proper distance may be checked by gradually increasing it until no change in the result occurs.

```
Boundary Condition 1
  Target Boundaries = 1
  Potential 1 = Real 0.0
  Potential 2 = Real 0.0
End
```

Results

With the given computational mesh the problem is solved in a few seconds. With the 20 072 bilinear elements the heating efficiency is 16.9 W. The corresponding results are shown in Fig. 15.1.

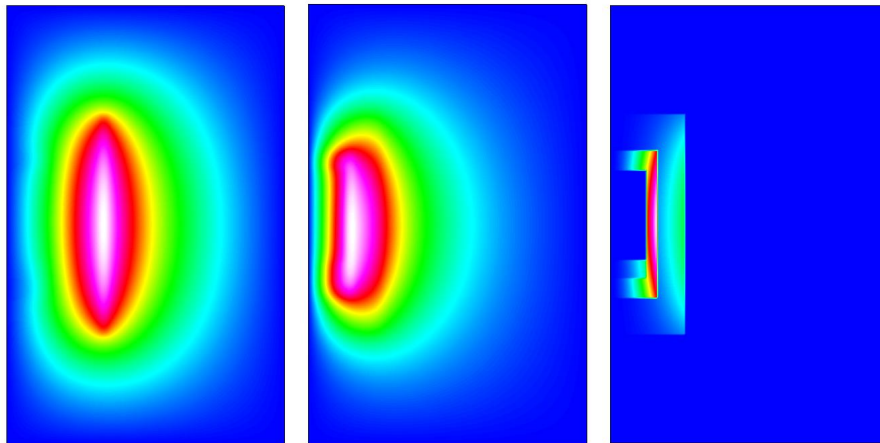


Figure 15.1: Induction heating of a simple crucible. a) in-phase component of the vector potential b) out-of-phase component of the vector potential c) Joule losses in the conductors

Tutorial 16

Fluid flow around an elastic beam

Directory: FluiStructureBeam

Solvers: ElasticSolve, FlowSolve, MeshSolve

Tools: ElmerGrid, editor, Fortran 90 compiler

Dimensions: 2D, steady-state

Case definition

A homogenous, elastic beam (Ω_2) is in a fluid flow (which takes place in the region Ω_1), see figure 16.1. The beam is 5.0 m long and it is rigidly supported on the end Γ_6 . At the incoming boundary Γ_1 , the fluid velocity in the x-direction is a parabola which reaches its maximum value 1 m/s at the point $y=5.0$ m. At the incoming and outgoing boundaries Γ_1 and Γ_2 velocities in the y-direction are 0 m/s. At the outgoing boundary the pressure is 0 Pa. The velocities on boundaries Γ_3 , Γ_4 and Γ_5 are 0 m/s. The fluid is incompressible and has density 1 kg/m, dynamic viscosity 0.5 kg/ms and Poisson ratio 0.3. Material properties for the beam are the density 1000 kg/m, Poisson ratio 0.3 and Young's modulus 1.0e3 N/m. The problem is to solve the maximum displacement of the beam and the pressure change in Ω_1 .

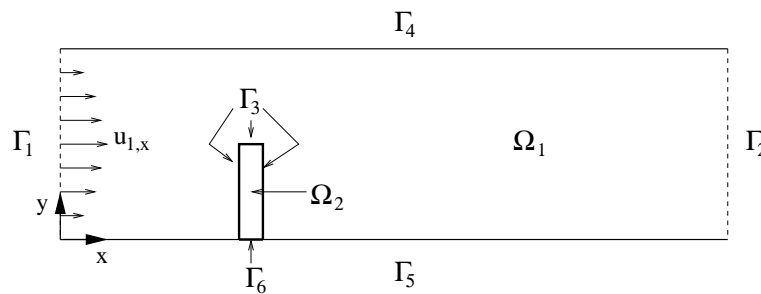


Figure 16.1: Geometry of the problem

The flow generates normal and shear stresses to the beam boundary Γ_3 . These forces deform the beam and this changes the flow. This coupled problem can be modelled using the Navier-Stokes and elasticity equations.

For the incompressible fluid the Navier-Stokes equations can be written as

$$\begin{aligned}
 -\nabla \cdot (2\mu\bar{\varepsilon}) + \rho\vec{u} \cdot \nabla\vec{u} + \nabla p &= 0 && \text{in } \Omega_1 \\
 \nabla \cdot \vec{u} &= 0 && \text{in } \Omega_1 \\
 \vec{u}_x &= (10y - y^2)/25 && \text{on } \Gamma_1 \\
 \vec{u}_x &= 0 && \text{on } \Gamma_i, i = 3, 4, 5 \\
 \vec{u}_y &= 0 && \text{on } \Gamma_i, i = 1, \dots, 5,
 \end{aligned} \tag{16.1}$$

where μ is the dynamic viscosity, $\bar{\varepsilon}$ is the strain tensor, ρ is the density, \vec{u} is the velocity vector and p is the pressure. It is assumed that the density and viscosity are constants.

For the homogeneous, elastic beam the elasticity equations are

$$\begin{aligned} -div[(I + \nabla u)\Sigma] &= 0 && \text{in } \Omega_2 \\ \Sigma &= \lambda(tr E)I + 2\mu E && \text{in } \Omega_2 \\ E &= \frac{1}{2}(\nabla u^T + \nabla u + \nabla u^T \nabla u) && \text{in } \Omega_2 \\ u &= 0 && \text{on } \Gamma_6 \\ (I + \nabla u)\Sigma n &= q && \text{on } \Gamma_3 \end{aligned} \quad (16.2)$$

where u is the displacement vector, Σ is the second Piola-Kirchhoff stress tensor, λ and μ are the Lamé constants and E is the Green-St Venant strain tensor, q is the surface traction and n is the unit normal to the beam boundary. The edge load q is calculated from the stresses obtained using the Navier-Stokes equations.

For updating the fluids mesh a linear elasticity equation is used. In mathematical form it can be written as,

$$\begin{aligned} -div\{\lambda tr[\varepsilon(v)]I + 2\mu\varepsilon(v)\} &= 0 && \text{in } \Omega_1 \\ v &= u|_{\Gamma_3} && \text{on } \Gamma_3 \\ v &= 0 && \text{on } \Gamma_i, i = 1, 2, 4, 5 \end{aligned} \quad (16.3)$$

where λ and μ are the Lamé constants, ε is linear strain tensor and v is the displacement vector. Here u is the solution to the elasticity equations.

Solution procedure

In this problem geometry and mesh is created by using ElmerGrid by the command

```
ElmerGrid 1 2 fsi.grd
```

In this problem, external Fortran functions are used to express a parabolic velocity inflow profile and to define a variable stiffness for the deforming mesh. The stiffness of the mesh is controlled by the Youngs modulus. In the case of deforming mesh, the absolute value of the parameter is not important but its changes over the domain are influential. The following function defines the mesh to be stiffer near the corner of the beam where the mesh is deformed most. The idea is to avoid too irregularly shaped elements in that area.

```
FUNCTION Youngs( Model, n, x ) RESULT( s )
  USE Types
  IMPLICIT NONE

  TYPE(Model_t) :: Model
  INTEGER :: n
  REAL(KIND=dp) :: x, s, xx, yy

  xx = Model % Nodes % x(n)
  yy = Model % Nodes % y(n)

  s = 1.0d0 / SQRT( (xx-11.0)**2 + (yy-4.9)**2 )

END FUNCTION Youngs
```

The following function is used to give the inflow boundary condition.

```
FUNCTION InFlow( Model, n, x ) RESULT( vin )
  USE Types
  IMPLICIT NONE

  TYPE(Model_t) :: Model
  INTEGER :: n
```

```

REAL(KIND=dp) :: yy,xx,x,vin,v0,vt

xx = Model % Nodes % x(n)
yy = Model % Nodes % y(n)

v0 = -(yy**2)+10*yy)/25

IF(x < 8.0) THEN
  vt = x/8.0
ELSE
  vt = 1.0
END IF

vin = v0*vt

END FUNCTION InFlow

```

In general, the user may supply his/hers own functions for defining general dependencies of variables or parameters. The function header should read as follows

```
FUNCTION FunctionName( Model, n, x ) RESULT( res )
```

where `Model` is a structure holding various information of the problem and `n` is the number of the node being considered. These are automatically given by the program. The parameter `x` is the value of the variable, which user defines to be given for the function as input. The variable is defined in the `sif` file. Finally, the function returns the result `res` for the parameter in consideration.

If the Elmer environment is successfully setup the compilation command should look like

```
elmerf90 -o FsiStuff FsiStuff.f90
```

The functions may be compiled also locally on a PC either for Linux or WindowsNT, provided that a Fortran90 compiler and the Elmer software are installed.

The next step is to edit the solver input file (`sif` file) with a text editor, *e.g.* Emacs.

- The Header section, where the mesh is defined.

```

Header
  Mesh DB "." "fsi"
End

```

- The Constants section need not contain any constant this time.

```

Constants
End

```

- The Simulation section gives the control data for the case.

```

Simulation
  Coordinate System = Cartesian 2D
  Simulation Type = Steady State
  Steady State Max Iterations = 50
  Steady State Min Iterations = 2
  Output Intervals = 1
  Output File = "fsi.result"
  Post File = "fsi.ep"
End

```

- The Body section is used to define which equations and materials are used for the simulation bodies. (Ω_2).

```
Body 1
  Equation = 1
  Material = 1
End
```

```
Body 2
  Equation = 2
  Material = 2
End
```

- The Material section defines the material properties of each body. Also the procedure Youngs is used.

```
Material 1
  Density = 1.0
  Viscosity = 0.5
  Poisson Ratio = 0.3
  Youngs Modulus = Variable time
  Real Procedure "FsiStuff" "Youngs"
End
```

```
Material 2
  Density = 1000
  Youngs Modulus = 1.0e3
  Poisson Ratio = 0.3
End
```

- The Solver section defines various control parameters for each solver.

```
Solver 1
  Equation = Navier-Stokes
  Stabilize = True
  Linear System Solver = Iterative
  Linear System Iterative Method = BiCGStab
  Linear System Preconditioning = ILU1
  Linear System Max Iterations = 500
  Linear System Convergence Tolerance = 1.0e-8
  Nonlinear System Max Iterations = 10
  Nonlinear System Convergence Tolerance = 1.0e-5
  Nonlinear System Newton After Tolerance = 1.0e-5
  Nonlinear System Newton After Iterations = 20
  Nonlinear System Relaxation Factor = 1.0
  Steady State Convergence Tolerance = 1.0e-4
End
```

```
Solver 2
  Equation = Elasticity Solver
  Variable = Displacement
  Variable DOFs = 2
  Procedure = "ElasticSolve" "ElasticSolver"
  Linear System Solver = Direct
  Linear System Direct Method = Banded
```

```

Nonlinear System Newton After Tolerance = 1.0e-3
Nonlinear System Newton After Iterations = 20
Nonlinear System Max Iterations = 100
Nonlinear System Convergence Tolerance = 1.0e-5
Nonlinear System Relaxation Factor = 1.0
Steady State Convergence Tolerance = 1.0e-4
End

```

```

Solver 3
Equation = Mesh Update
Linear System Solver = Iterative
Linear System Iterative Method = BiCGStab
Linear System Preconditioning = ILU1
Linear System Max Iterations = 500
Linear System Convergence Tolerance = 1.0e-8
Steady State Convergence Tolerance = 1.0e-4
End

```

- The Equation section. This section defines the equations for a body. Plane stress model for the elastic beam is applied. Setting this keyword to False enables the plane strain assumption.

```

Equation 1
Active Solvers(2) = 1 3
End

```

```

Equation 2
Active Solvers = 2
Plane Stress = True
End

```

- In the Boundary Condition section different boundary conditions are defined. In Boundary Condition 1 the shape of inflow is defined with the Fortran procedure InFlow. Note also the bc 3, where keyword FSI BC is supplied to define the boundary on which the fluid affects a force on the elastic beam. Also, on the same boundary, the mesh is constraint to follow the deformation of the beam by the two Equals lines.

```

Boundary Condition 1
Target Boundaries = 1
Velocity 1 = Variable Time
Real Procedure "FsiStuff" "InFlow"

Velocity 2 = 0.0
Mesh Update 1 = 0.0
End

```

```

Boundary Condition 2
Target Boundaries = 2
Velocity 2 = 0.0
Pressure = 0.0
Mesh Update 1 = 0.0
End

```

```

Boundary Condition 3
Target Boundaries = 3
Velocity 1 = 0.0

```



```
Velocity 2 = 0.0
FSI BC = True
Mesh Update 1 = Equals Displacement 1
Mesh Update 2 = Equals Displacement 2
End

Boundary Condition 4
  Target Boundaries(2) = 4 5
  Velocity 1 = 0.0
  Velocity 2 = 0.0
  Mesh Update 2 = 0.0
End

Boundary Condition 5
  Target Boundaries = 6
  Displacement 1 = 0.0
  Displacement 2 = 0.0
End
```

Results

After the solution is done, view the results with ElmerPost. Read the result file fsi.ep in ElmerPost by choosing Read Model File. The fsi.ep file is located in the same directory as the mesh files.

Table 16.1: Max. displacement and pressure difference

Elements	max $ u $ [m]	max Δp [Pa]
540	1.34	6.36

As a result the maximum absolute value of displacement and the pressure difference is given (see table 16.1).

Tutorial 17

Thermal actuator driven with electrostatic currents

Directory: ThermalActuator

Solvers: StatCurrentSolve, HeatSolve, StressSolve

Tools: ElmerGrid, editor

Dimensions: 3D, Steady-state

Case definition

The tutorial introduces a micro mechanical thermal actuator as shown in Fig. 17.1. A static electric current is driven through the actuator. The power loss due to the resistance of the actuator is transformed into heat which in turn causes thermal stresses into the structure. The electric current thus results in deformation of the actuator. In industry, such an actuator might be used to control the position of a micromechanical component.

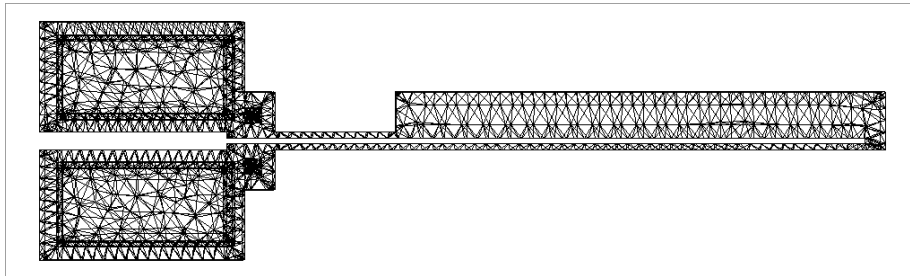


Figure 17.1: The geometry of the actuator.

Solution procedure

The problem is solved by first iterating the electrostatic current solver and heat equation until both are converged. The temperature distribution is then used as a load for stress analysis solver which calculates the actual deformation of the structure. The electric conductivity of the actuator depends on the temperature and thus the electrostatic - thermal problem is coupled in both directions.

The computational mesh for this particular tutorial is created by using Ansys software. The details of the mesh are written into files called `ExportMesh` by a certain Ansys macro and converted to Elmer format by the ElmerGrid program. The command to use is

```
ElmerGrid 4 2 ExportMesh -order 1.0 0.1 0.001 -o thermal
```

The above command reads in the Ansys mesh files, arranges the mesh nodes in a reasonable way and saves the mesh in Elmer format in a directory called `thermal`.

The geometry of the problem includes only one body and material. Boundary conditions are defined on the actuator legs, which are kept at constant electric potential, temperature and position. Thus, only Dirichlet boundary conditions are used.

The header and simulation blocks of the solver input file are

```
Header
  Mesh DB "." "thermal"
End

Simulation
  Coordinate System = Cartesian 3D
  Simulation Type = Steady State
  Steady State Max Iterations = 30
  Output Intervals = 1
  Output File = "actuator.result"
  Post File = "actuator.ep"
End
```

An initial condition for temperature is defined in order to ease the convergence of the iterative solvers. Also, a body force for the heat equation solver defining the Joule heating is needed. These both have to be declared in the body section as follows:

```
Body 1
  Equation = 1
  Material = 1
  Initial Condition = 1
  Body Force = 1
End
```

The solution procedure requires the use of three solvers: Static current solver, heat equation solver and the stress analysis solver. The equation block below defines that these solvers are used.

```
Equation 1
  Active Solvers(3) = Integer 1 2 3
  Calculate Joule Heating = True
End
```

The solver blocks define the parameters of the respecting solvers. The static current conduction problem is tackled by an iterative conjugate gradient method (CG). For heat equation, a stabilized biconjugate gradient method is used. The coupled problem of these two solvers is difficult since the static current calculated heats the structure on each step, and the rise of temperature makes the current conduction more and more difficult. To overcome this problem, a relaxation factor of 0.5 is defined for the heat equation solver.

```
Solver 1
  Equation = Stat Current Solver
  Procedure = "StatCurrentSolve" "StatCurrentSolver"
  Variable = Potential
  Variable DOFs = 1
  Calculate Volume Current = True
  Calculate Electric Conductivity = True
  Linear System Solver = Iterative
  Linear System Iterative Method = CG
  Linear System Preconditioning = ILU3
  Linear System Max Iterations = 300
```

```

Linear System Convergence Tolerance = 1.0e-8
Nonlinear System Max Iterations = 1
Nonlinear System Convergence Tolerance = 1.0-6
Nonlinear System Newton After Iterations = 3
Nonlinear System Newton After Tolerance = 1.0e-12
Nonlinear System Relaxation Factor = 1.0
Steady State Convergence Tolerance = 1.0e-6
End

```

```

Solver 2
Equation = Heat Equation
Variable = Temperature
Variable DOFs = 1
Linear System Solver = Iterative
Linear System Iterative Method = BiCGStab
Linear System Preconditioning = ILU1
Linear System Max Iterations = 350
Linear System Convergence Tolerance = 1.0e-9
Nonlinear System Max Iterations = 1
Nonlinear System Convergence Tolerance = 1.0e-07
Nonlinear System Newton After Iterations = 3
Nonlinear System Newton After Tolerance = 1.0e-12
Nonlinear System Relaxation Factor = 0.5
Steady State Convergence Tolerance = 1.0e-07
End

```

For stress analysis, a direct solver is used instead of an iterative solver. It is often difficult for the iterative solver to find a solution for a structure that contains parts with varying stiffness properties, which is obviously the case here (try the iterative solver and see!). The stress analysis solver is called first only after the coupled iteration of two previous solvers is complete. This is possible since the deformation of the structure is so small that it does not change the current density distribution. Defining stress analysis this way saves computational time. It is possible to iterate all the three solvers until convergence by commenting the Exec Solver line.

```

Solver 3
Exec Solver = After All
Equation = Stress Analysis
Variable = Displacement
Variable DOFs = 3
Linear System Solver = Direct
Linear System Direct Method = Banded
Nonlinear System Max Iterations = 1
Nonlinear System Convergence Tolerance = 1.0e-6
Nonlinear System Newton After Iterations = 3
Nonlinear System Newton After Tolerance = 1.0e-12
Nonlinear System Relaxation Factor = 1.0
Steady State Convergence Tolerance = 1.0e-6
End

```

The material of the structure has a temperature dependent electric conductivity. This, as well as other material parameters, is defined in the material block. Note that a MEMS unit system is used.

```

Material 1
Electric Conductivity = Variable Temperature
Real

```

```
298.0 4.3478e10
498.0 1.2043e10
698.0 5.1781e9
898.0 2.7582e9
1098.0 1.6684e9
1298.0 1.0981e9
1683.0 1.0
2000.0 1.0
End
```

```
Density = 2.3e-15
Heat Conductivity = 32.0e6
Youngs Modulus = 169.0e3
Poisson Ratio = 0.22
Heat Expansion Coefficient = 2.9e-6
Reference Temperature = 298.0
End
```

Finally, the initial condition, thermal heat load for stress analysis, and the boundary conditions are defined.

```
Initial Condition 1
  Temperature = 298.0
End
```

```
Body Force 1
  Heat Source = Equals Joule Heating
End
```

```
Boundary Condition 1
  Target Boundaries = 1
  Potential = 0
  Temperature = 298
  Displacement 1 = 0.0
  Displacement 2 = 0.0
  Displacement 3 = 0.0
End
```

```
Boundary Condition 2
  Target Boundaries = 2
  Potential = 7
  Temperature = 298
  Displacement 1 = 0.0
  Displacement 2 = 0.0
  Displacement 3 = 0.0
End
```

Results

The problem converges after 27 steady state iterations on the tolerance limits defined above. The calculation takes about 180 cpu seconds of which 40 cpus is spent in solving the stress analysis equation. The calculations were performed on a Compaq Alpha Server with a 1 GHz central processor.

Result for temperature distribution and the displacement are shown in Figs 17.2 and 17.3. The temperature rises unrealistically high in this example because all heat transfer mechanisms out of the structure are

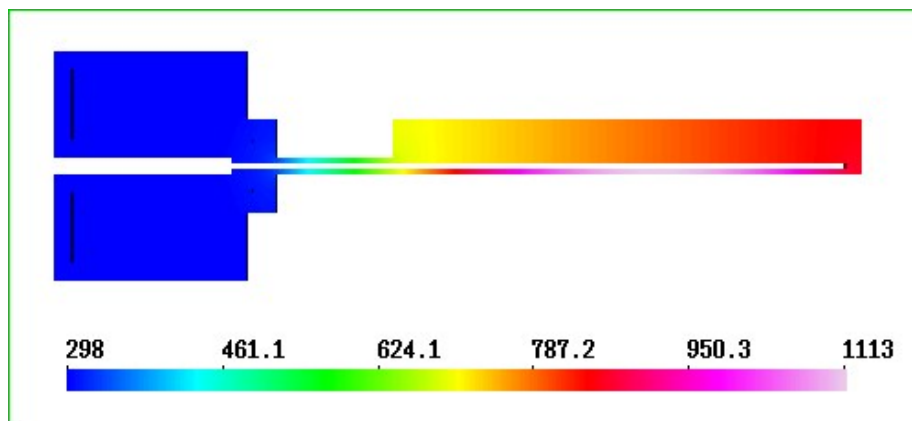


Figure 17.2: Temperature distribution.

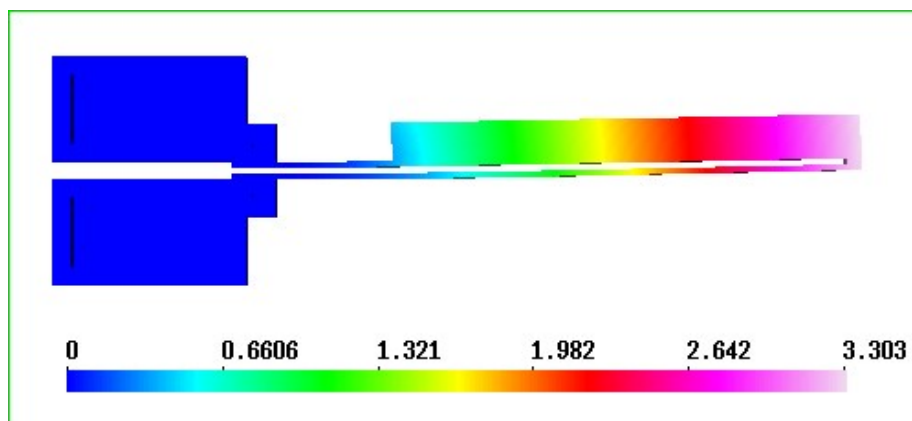


Figure 17.3: The displacement of the actuator.

neglected. Presumably at least the heat radiation is of major importance in this case. For displacement, the results show a movement of about 3.3 micrometers for the actuator tip.

Tutorial 18

Axisymmetric coating process

Solvers: FlowSolve, FreeSurfaceReduced

Tools: ElmerGrid, editor

Dimensions: 2D, Steady-state

Case definition

The optical fibers are quite fragile and must therefore be coated with a layer of polymer before they are stored. This means that the coating process must be done with the same speed as the drawing of optical fibers. When the diameter of the fiber is only $125\ \mu\text{m}$ this sets high demands for the coating geometry since it must provide even coating at high draw speeds. In Elmer a tailored free surface boundary condition allows an efficient solution of this particular problem.

Solution procedure

The mesh is done with ElmerGrid in the directory coat by the command

```
ElmerGrid 1 2 coat.grd
```

Therefore the header reads

```
Header
  Mesh DB "." "coat"
End
```

The geometry is axisymmetric and the problem is solved in steady state. Typically around 10 iterations is needed to solve the problem but to be on the safe side 30 is set as the maximum.

```
Simulation
  Coordinate System = Axi Symmetric
  Simulation Type = Steady State
  Steady State Max Iterations = 30
  Output Intervals = 1
  Output File = "coat.result"
  Post File = "coat.ep"
End
```

In this case there is only one body which comprises of the polymer floating between the coating cup and the optical fiber.

```
Body 1
  Equation = 1
  Material = 1
End
```

The presented solution used four different solvers. The Navier-Stokes solver is required to solve the flow field for the polymer.

```
Solver 1
  Equation = Navier-Stokes
  Stabilize = True
  Internal Move Boundary = Logical False
  Nonlinear System Max Iterations = 5
  Nonlinear System Convergence Tolerance = 1.0e-7
  Nonlinear System Newton After Iterations = 2
  Nonlinear System Newton After Tolerance = 1.0e-2
  Nonlinear System Relaxation Factor = 0.7
  Linear System Solver = Iterative
  Linear System Iterative Method = BiCGStab
  Linear System Preconditioning = ILU1
  Linear System Max Iterations = 100
  Linear System Convergence Tolerance = 1.0e-10
  Steady State Convergence Tolerance = 1.0e-7
End
```

A tailored free surface solver is used to find the position of the free surface with a given flow field. The variable being solved is the displacement of the free surface. Relaxation is used to avoid over-shooting during the iteration. This solver does not solve any matrix equations. Instead it solves the radius from the mass conservation constraint for each node on the free surface separately. There is a possibility to do the mapping also within the solver using a 1D scheme but this is disabled by setting the `Perform Mapping` to be `False`.

```
Solver 2
  Equation = "Free Surface Reduced"
  Procedure = "FreeSurfaceReduced" "FreeSurfaceReduced"
  Variable = Dx
  Variable DOFs = 1
  Nonlinear System Relaxation Factor = 0.7
  Nonlinear System Convergence Tolerance = 1.0e-3
  Steady State Convergence Tolerance = 1.0e-3
  Perform Mapping = Logical False
End
```

The mesh update solver is required to map the computational mesh so that it corresponds to the altered geometry. Here the displacements of the free surface have already been computed and this solver solves the displacements inside the domain. Note that solvers 1, 2 and 3 are coupled and therefore the system must be solved iteratively

```
Solver 3
  Equation = Mesh Update
  Linear System Solver = Iterative
  Linear System Iterative Method = BiCGSTAB
  Linear System Preconditioning = ILU
  Linear System Convergence Tolerance = 1.0e-12
  Linear System Max Iterations = 200
  Linear System Symmetric = True
  Steady State Convergence Tolerance = 1.0e-4
End
```

In the end, an additional solver is used to compute the forces acting on the fiber. This does not affect the results.


```
Solver 4
  Equation = Fluidic Force
  Procedure = "FluidicForce" "ForceCompute"
  Calculate Viscous Force = Logical True
End
```

Additionally there are two solvers for saving the results in a form that is more useful than plain pictures. The `SaveScalars` saves the scalar values, such as the diameter and force values, and the `SaveLine` saves the free surface.

```
Solver 5
  Equation = SaveScalars
  Procedure = "SaveData" "SaveScalars"
  Filename = "scalars.dat"
End
```

```
Solver 6
  Equation = SaveLine
  Procedure = "SaveData" "SaveLine"
  Filename = "kurvi.dat"
End
```

The equation includes only the solvers that need a permutation vector pointing out the active nodes. Therefore the save utilities do not need to belong to the set of active solvers.

```
Equation 1
  Active Solvers(4) = 1 2 3 4
End
```

The material parameters are those of the polymer. Additionally elasticity parameters are needed because the solver that updates the mesh is actually a linear elasticity solver.

```
Material 1
  Density = 1.0
  Viscosity = 1.0
  Poisson Ratio = 0.3
  Youngs Modulus = 1.0
End
```

Five different boundary conditions are needed. The origin is a symmetry axis and therefore the radial velocity is set to zero. The axial velocity is the draw velocity.

```
Boundary Condition 1
  Name = "Symmetry"
  Target Boundaries = 1
  Velocity 2 = -10.0      ! The draw velocity
  Velocity 1 = 0.0
  Compute Fluidic Force = Logical True
  Mesh Update 1 = 0.0
End
```

The free surface has a condition stating that the reduced order free surface solver should be solved for that. Additionally the free surface is a boundary condition for the mesh update, and a line to be saved.

```
Boundary Condition 2
  Name = "Free"
  Target Boundaries = 2
  Mesh Update 1 = Equals Dx
```

```
Mesh Update 2 = 0.0
Free Surface Reduced = Logical True
Save Line = Logical True
End
```

At the outlet the radial velocity should vanish and the axial coordinate should be fixed.

```
Boundary Condition 3
Name = "Outlet"
Target Boundaries = 3
Velocity 1 = 0.0
Mesh Update 2 = 0.0
End
```

At the inlet it is assumed that there is no radial velocity and that the pressure acting on the surface is zero.

```
Boundary Condition 4
Name = "Inlet"
Target Boundaries = 4
Velocity 1 = 0.0
Pressure = 0.0
Mesh Update 2 = 0.0
End
```

Finally, no-slip conditions are set for the boundaries with the walls of the coater.

```
Boundary Condition 5
Name = "No-slip"
Target Boundaries = 5
Velocity 1 = 0.0
Velocity 2 = 0.0
Mesh Update 1 = 0.0
Mesh Update 2 = 0.0
End
```

Results

In the given case solution is obtained after 13 iterations. The solution gives the final radius, the forces, and the profile of the free surface. To visualize the true free surface you may do the following. Read in the only the last timestep and in ElmerPost give the following commands:

```
math nodes0 = nodes
math nodes = nodes0 + Mesh.Update
```

Note that this does not work if there is more than one set of variable values.

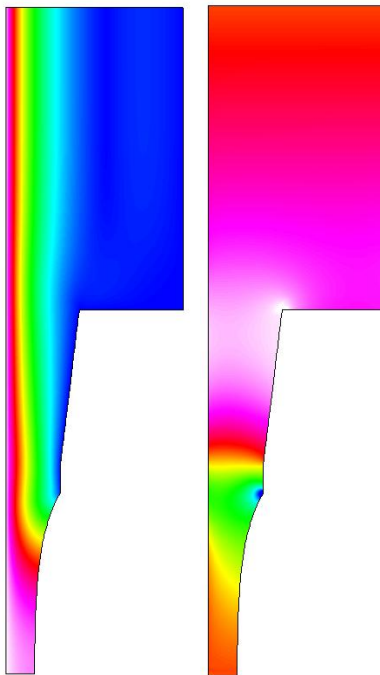


Figure 18.1: The velocity and pressure fields in a simple coating geometry. The solution utilizes the reduced dimensional free surface solver.

Tutorial 19

Blood ejection from a ventricle into aorta

Directory: ArteryFlow

Solvers: FlowSolve,ElasticSolve, OutletCompute

Tools: Editor, Fortran 90 compiler, ElmerGrid

Dimensions: 2D, Transient

Case description

This tutorial is about simulating blood ejection in to the elastic human aorta. The idea is to mimic left ventricle contraction and resulting pulse propagation in an elastic conduit. In the simulation about 0.8 desiliters of blood is ejected to a 50 cm long elastic aorta during a time period of 400 ms. In order to get the outlet of the model behave physiologically more realistic way, a one dimensional model is coupled with the higher order model.

Solution procedure

First we generate the mesh of 366 eight-node quadrilaterals elements with the command

```
ElmerGrid 1 2 contra
```

Next we generate one dimensional mesh to the outlet of the 2D model. The program `AddOneDim` is posed to be run in the mesh directory `contra`. The length, the number of the elements, and the coordinate direction of the 1D section will be asked.

In the simulation block the timestep is set equal to 1 ms and total simulation time equal to 600 ms. The geometry consists of five bodies of which the first three are for the fluid volume. Body number 1 os the contracting volume. Body 2 is a short rigid channel between the body 1 and the elastic artery. Artificial compressibility method is used for the fluid volume (body 3) which is in contact with the elastic wall (body 4). One dimensional model is the body 5. Material settings for those are following:

```
! Bodies 1 and 2 (blood)
Material 1
  Density = 1000
  Viscosity = 3.5e-3
  Youngs Modulus = 1
  Poisson Ratio = 0.3
End

! Body 3 (blood)
```

```

Material 2
  Density = 1000
  Viscosity = 3.5e-3
  Youngs Modulus = 1
  Poisson Ratio = 0.3
  Compressibility Model = Artificial Compressible
  Artificial Compressibility = 3.3E-5
End

! Body 4 (elastic wall)
Material 3
  Density = 1010
  Youngs Modulus = 3.0e5
  Poisson Ratio = 0.45
End

! One dimensional model
Material 4
  Density = 1010.0
  Artery Wall Youngs Modulus = Real 3.0e5
  Artery Radius = Real 0.0135
  Artery Wall Thickness = Real 0.002
  Artery Poisson Ratio = Real 0.45
End

```

Notice that the radius of the one dimensional model (Artery Radius) is to the midplane of the wall (inner radius + half of the wall thickness). The overall FSI iteration scheme is started by one dimensional solver (OutletCompute, see the solver manual), after that Navier-Stokes, elasticity and mesh update solvers are run. Steady state convergence tolerance is set equal to 1.0E-4 for each of the solvers. The nonlinearities of each of the solvers are computed within the FSI scheme loop, that is, the flag Nonlinear System Max Iterations is set equal to 1. Artificial compressibility coefficient is computed by the equation $c = (1 - \nu^2)[D/(E h)]$, where ν is the Poisson ratio of the artery wall, D , E and h are the inner diameter, Young's modulus and the thickness of the artery, respectively.

The only driving force of the system, the wall motion of the contracting fluid domain is given by the fortran function Motion, see the figure 19.1. The boundary condition setting is

```

! Moving boundary
Boundary Condition 1
  Target Boundaries = 1
  Velocity 1 = 0
  Velocity 2 = Equals Mesh Velocity 2
  Mesh Update 1 = Real 0
  Mesh Update 2 = Variable Time
  Real Procedure "./Motion" "Motion"
End

```

At the outlet, the pressure boundary condition is given by the function OutletPres and the corresponding radial displacement of the end wall of the outlet is given by the function OutletdX

```

! Outlet pressure of the 2D model
Boundary Condition 2
  Target Boundaries = 2
  Flux Integrate = Logical True
  Flow Force BC = True
  Pressure 2 = Variable Time
  Real Procedure "./ArteryOutlet" "OutletPres"

```

```
Mesh Update 2 = Real 0
End
```

```
! Radial displacement of the end wall at the outlet of 2D model
Boundary Condition 9
  Target Boundaries = 9
  Displacement 1 = Variable Time
    Real Procedure "ArteryOutlet" "OutletdX"
  Displacement 2 = 0
End
```

FSI interface boundary is described as following

```
! FSI interface boundary
Boundary Condition 11
  Target Boundaries = 11
  Velocity 1 = Equals Mesh Velocity 1
  Velocity 2 = Equals Mesh Velocity 2
  Mesh Update 1 = Equals Displacement 1
  Mesh Update 2 = Equals Displacement 2
  Force BC = Logical True
End
```

Finally, the coupling of the 1D model with the 2D is done at the inlet boundary as

```
Boundary Condition 16
  Target Boundaries = 16
  Fluid Coupling With Boundary = Integer 2
  Structure Coupling With Boundary = Integer 9
End
```

Results

The contraction is curve seen in the figure 19.1 and the velocity fields at different time levels are presented in the figure 19.2. Postprocessing instructions are given in the file `PostProcessingInstr.txt`.

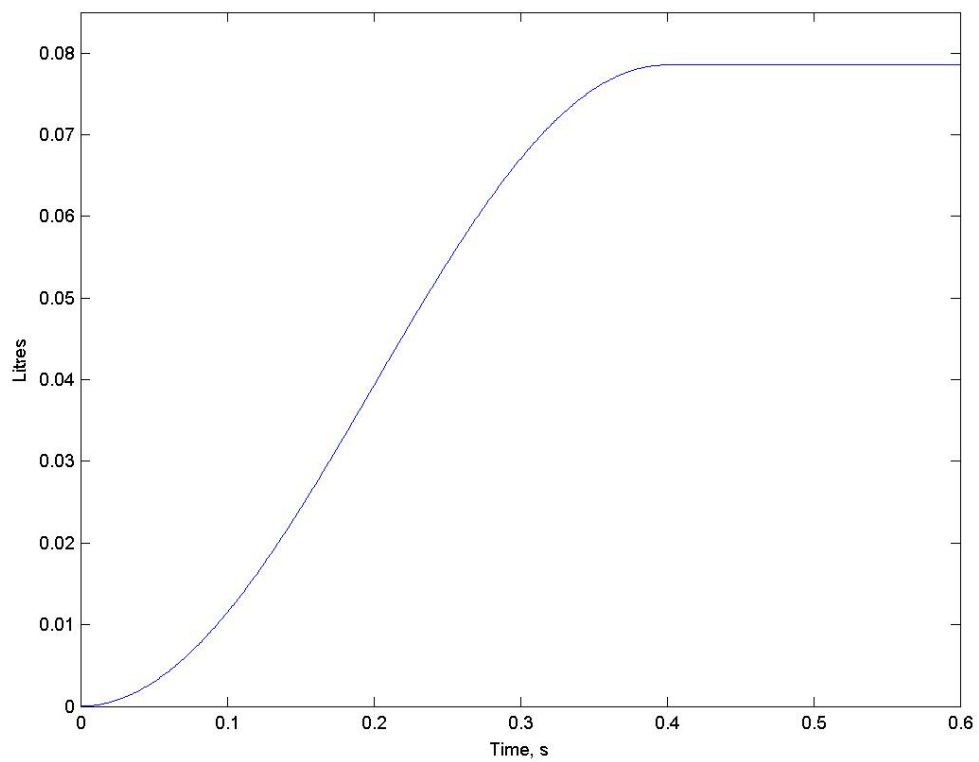


Figure 19.1: Contraction curve generated by the function `Motion`.

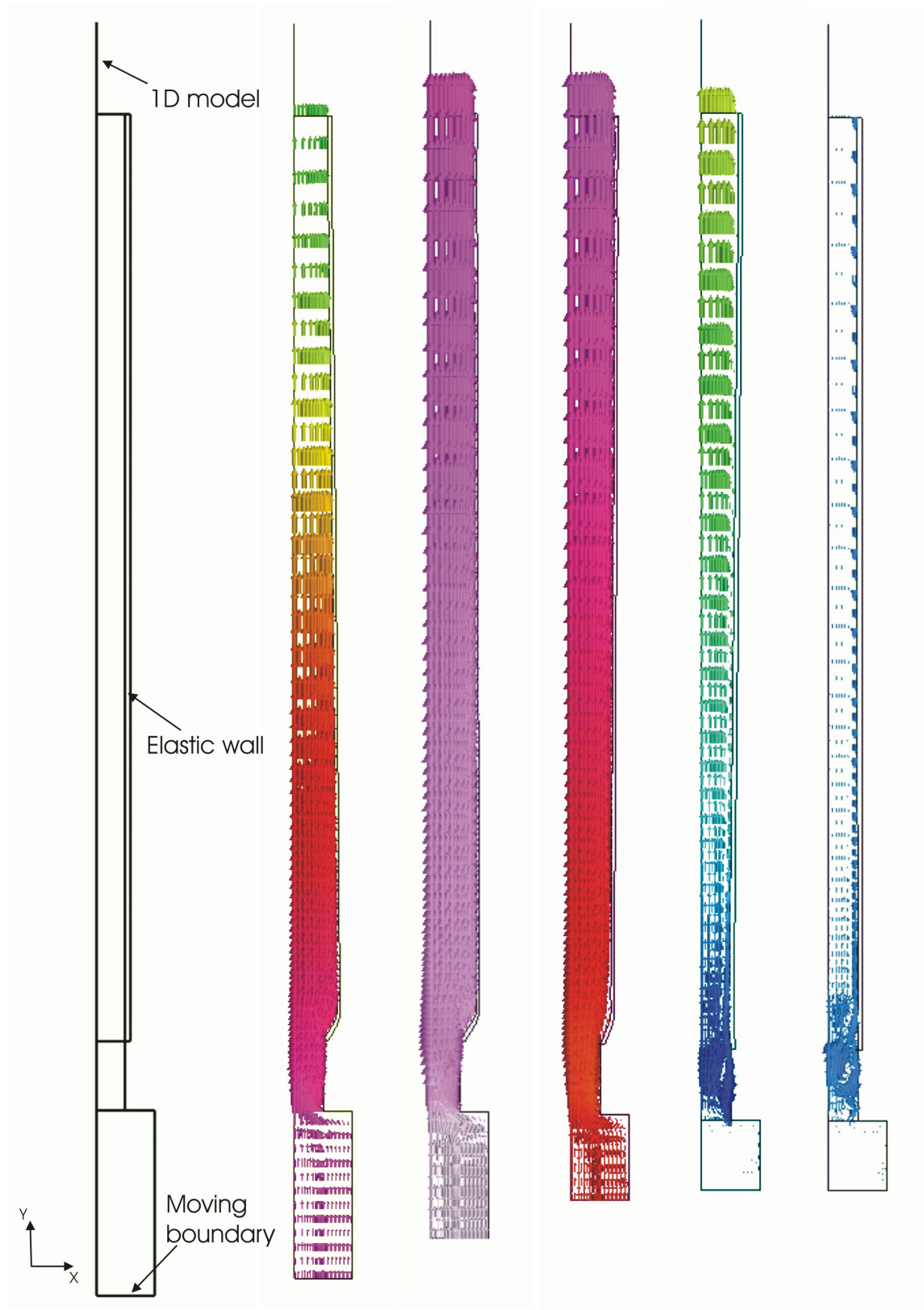


Figure 19.2: The geometry of the model and velocity fields at 5 time steps, 100, 200, 300, 400 and 500 ms. The displacements of the wall are magnified by factor of 10.

Part III

Utility Problems

Tutorial 20

Operator splitting in the evolutionary heat equation

Directory: TemperatureOperatorSplitting

Solvers: HeatSolve, TransportEquationSolver, RateOfChangeSolver

Tools: Editor

Dimensions: 2D, Transient simulation

Introduction

The drawback of the stabilized finite element formulations available in Elmer to solve the convection-diffusion equation and Navier-Stokes equations is that these methods are computationally expensive, in particular when the residual-free-bubbles formulation is used. In evolutionary problems the reduction of computational cost may be attempted by applying operator splitting techniques in which the original equation at each time step is splitted up into subproblems that are easier to solve numerically.

The aim of this chapter is to provide an illustrative example on using operator splitting capability in the solution of the time-dependent heat equation. For the theoretical background of the operator splitting scheme applied here the reader is referred to Elmer Models Manual and references therein.

Case description

The problem considered here is the solution of the time-dependent heat equation in the homogeneous L-shaped plate the geometry of which is shown in Figure 27.1. The density of the material is taken to be unity, while the heat conductivity k of the material is taken to be a parameter with values ranging between 0.05 and 0.01. The plate is heated by a constant internal heat source magnitude of which equals to unity. The convection velocity field is assumed to be constant with the two Cartesian components of the velocity vector equaling to unity. The boundaries Γ_i , $i = 1, 4, 5, 6$, are kept at constant temperature 0, while the boundaries Γ_2 and Γ_3 are assumed to be insulated, i.e. the heat flux on this part of the boundary vanishes.

The problem is to solve the temperature distribution in the plate. The time interval for the simulation is taken to be $[0,2]$ and the temperature of the plate is 0 at the time $t = 0$.

Solution procedure

Using operator splitting the solution of the heat equation may be replaced at each time step by the solution of three subproblems consisting of two time-dependent Poisson equations and the convective transport problem. The effects of diffusion and convection are decoupled by this splitting so that the diffusion and convection phenomena are taken into account by the steps involving the solution of the Poisson equation and the convective transport problem, respectively.

The time-dependent Poisson equations can be solved using the basic heat equation solver in Elmer. To avoid the use of stabilized finite element formulations in the solution of the convective transport problem this subproblem is solved by discretizing an equivalent wave-like equation formulation (second order in time equation).

When the operator splitting method is applied, specific care is needed in prescribing boundary and initial conditions for the simulation. While the boundary conditions (and initial conditions) for the steps involving the solution of the Poisson equation may be defined in the usual manner, the boundary conditions for the convective transport problem may be prescribed only on the part of inflow boundary on which the temperature is prescribed. In the example the inflow boundary is the union of the boundary segments $\Gamma_i, i = 1, 2, 5, 6$, so the boundary conditions for the convective transport equation should be given on $\Gamma_i, i = 1, 5, 6$. In addition to prescribing the boundary conditions, the rate of change of the field subject to the convection operator (here temperature) is needed as an initial condition at the beginning of pure convection step. This field can be solved using a specific solver (`RateOfChangeSolver`) prior to the solution of the convective transport problem. The boundary conditions for this solver should be prescribed on the same part of the boundary on which the boundary conditions for the convective transport problem are prescribed.

It should be noted that in connection with the operator splitting method the user should specify an even number of time steps. Although the details of implementation need not be known in order to use the operator splitting ability, it is noted that the the running of two successive time steps actually constitutes a single operator splitting scheme step as described in Elmer Models Manual. There are thus three equations (referred to as `Heat Equation`, `Rate Of Change Equation` and `Transport Equation`) which may be solved at one time step. At odd time steps all these equations are solved meaning that both the diffusion and convection steps are taken, whereas at even time steps the solution of the convective transport problem is omitted so that the diffusion step is performed only. Physically meaningful results satisfying all the essential boundary conditions may thus be written to the results file only after even time steps.

The mesh for this example was created using Femlab and tools for converting meshes from Femlab to Elmer format. The mesh in Elmer format is given in the tutorial files. This unstructured mesh consists of 8458 elements each of which having three nodes.

The Header section of the solver input file is used to declare the directory containing the Elmer mesh files:

```
Header
  Mesh DB "." "femlab-mesh"
End
```

The Simulation section of the solver input file is used to declare the coordinate system and simulation type as well as simulation parameters related to the time discretization scheme:

```
Simulation
  Coordinate System = String "Cartesian 2D"
  Simulation Type = String "Transient"
  Timestepping Method = String "Crank-Nicolson"
  Timestep Intervals(1) = Integer 200
  Timestep Sizes(1) = Real 0.01
  Output Intervals(1) = Integer 1
  Steady State Max Iterations = Integer 1
  Post File = File "os-example.ep"
End
```

Here the keyword `Timestepping Method` is used to define the time discretization scheme that is used in the solution of the time-dependent Poisson equations. Note that one should not use multi-step methods in connection with the operator splitting method. The time discretization scheme that is used in the solution of the convective transport problem is fixed and need not be specified. It should be noted also that the number of time steps should be even. Since the equations solved at one time step are not coupled and can thus be solved in sequential manner, `Steady State Max Iterations` may be taken to be 1.

The Body section of the solver input file is used to declare integer identifiers for body forces, equations, initial conditions and materials:

```

Body 1
  Body Force = Integer 1
  Equation = Integer 1
  Initial Condition = Integer 1
  Material = Integer 1
End

```

The Equation section of the solver input file in turn has the following declaration

```

Equation 1
  Active Solvers(3) = Integer 1 2 3
End

```

indicating that three equations are to be solved using solvers with the integer identifiers 1, 2 and 3. Accordingly, three Solver sections are needed. The first Solver section is used for the Poisson equation and has the following declarations:

```

Solver 1
  Equation = String "Heat Equation"
  Variable = String "Temperature"
  Variable Dofs = Integer 1
  Linear System Solver = String "Iterative"
  Linear System Iterative Method = String "BiCGStab"
  Linear System Max Iterations = Integer 350
  Linear System Convergence Tolerance = Real 1.0e-08
  Linear System Abort Not Converged = Logical True
  Linear System Preconditioning = String "ILU0"
  Steady State Convergence Tolerance = Real 1.0e-05
  Stabilize = Logical False
  Bubbles = Logical False
End

```

Note that there is no need to use stabilization, so the values of the keywords `Stabilize` and `Bubbles` may be set to be `False` to reduce the computational cost.

The remaining two Solver sections are related to the convective transport problem. The first one of these sections is used to declare the solver parameters related to the problem (Rate Of Change Equation) the solution of which gives the approximation to the rate of change of the temperature at the beginning of pure convection step. The contents of this solver section is

```

Solver 2
  Equation = String "Rate Of Change Equation"
  Procedure = File "RateOfChange" "RateOfChangeSolver"
  Variable = String "Udot0"
  Variable Dofs = Integer 1
  Advection = String "Constant"
  Advection Variable = String "Temperature"
  Linear System Solver = String "Iterative"
  Linear System Iterative Method = String "BiCGStab"
  Linear System Max Iterations = Integer 350
  Linear System Convergence Tolerance = Real 1.0e-08
  Linear System Abort Not Converged = Logical True
  Linear System Preconditioning = String "ILU0"
  Steady State Convergence Tolerance = Real 1.0e-05
End

```

Here the keyword `Advection Variable` is used to declare the quantity which is subject to the convection operator, while the keyword `Advection` is used to define the type of the velocity vector. The name `Udot0` is used for the solution of this problem.

Finally, the `Solver` section for the wave-like equation, which is equivalent to the convective transport equation, has the following declarations:

```
Solver 3
  Equation = String "Transport Equation"
  Procedure = File "TransportEquation" "TransportEquationSolver"
  Time Derivative Order = Integer 2
  Variable = String "U"
  Variable Dofs = Integer 1
  Advection = String "Constant"
  Advection Variable = String "Temperature"
  Rate Of Change Equation Variable = String "Udot0"
  Linear System Solver = String "Iterative"
  Linear System Iterative Method = String "BiCGStab"
  Linear System Max Iterations = Integer 350
  Linear System Convergence Tolerance = Real 1.0e-8
  Linear System Abort Not Converged = Logical True
  Linear System Preconditioning = String "ILU0"
  Steady State Convergence Tolerance = Real 1.0e-05
End
```

The name `U` is used for the solution of the convective transport problem. The value of the keyword `Time Derivative Order` must be 2. The use of the keywords `Advection Variable` and `Advection` is similar to that explained in connection with the second solver section.

The `Material` section is used to declare the material properties and, as the velocity vector in the convection operator is of constant type, also the components of the velocity vector. In the case $k = 0.01$ the contents of this section is

```
Material 1
  Density = Real 1
  Heat Capacity = Real 1
  Heat Conductivity = Real 0.01
  Advection Velocity 1 = Real 1
  Advection Velocity 2 = Real 1
End
```

`Body Force` section is used to declare the body force in the Poisson equations.

```
Body Force 1
  Heat Source = Real 1
End
```

Finally, the initial conditions and boundary conditions are specified. The temperature at $t = 0$ is defined by giving the declaration

```
Initial Condition 1
  Temperature = Real 0
End
```

Two `Boundary Condition` sections are needed. The first one is used to prescribe the boundary conditions on the part of inflow boundary where the temperature is given:

Table 20.1: The maximum temperature at $t = 2.0$. For comparison the maximum temperature according to the steady state solution is also recorded.

Method	k		
	0.05	0.025	0.01
OS	1.0235	1.0424	1.0677
S	1.0269	1.0436	1.0418
S(steady state)	1.0279	1.0437	1.0418
RFB	1.0271	1.0458	1.0853
RFB(steady state)	1.0286	1.0462	1.1050

```
Boundary Condition 1
  Target Boundaries(3) = Integer 1 2 4
  Temperature = Real 0
  Udot0 1 = Real 0
  U 1 = Real 0
End
```

The rate of change of the temperature is zero on this part of the boundary as the temperature is kept fixed. Thus the zero boundary conditions for `Udot0` are defined. The boundary value of the solution to the convective transport problem should equal to the temperature. Therefore zero boundary conditions for `U` are also defined.

The second Boundary Condition section is used to define the Dirichlet boundary conditions on the outflow boundary:

```
Boundary Condition 2
  Target Boundaries(1) = Integer 3
  Temperature = Real 0
End
```

Note that the zero heat flux condition need not be specified explicitly. Similarly, the treatment of the outflow boundary conditions for the wave-like equation are handled automatically by the code and need not be specified.

Results

From a numerical point of view the solution of the problem becomes increasingly difficult as the heat conductivity k tends to zero. In order to examine possible dependence on the heat conductivity parameter the problem was solved in three cases with k taking values 0.05, 0.025 and 0.01. For comparison the same case was solved using three alternative methods. Here the operator splitting scheme is referred to as OS, S is the stabilized finite element method and RFB is the method based on the residual-free-bubbles formulation. In the case of methods S and RFB the simulation was performed using 100 time steps which corresponds to the number of 200 time steps used in the case of operator splitting scheme.

The maximum temperature at $t = 2.0$ is recorded in Table 20.1. The CPU time used in the simulation to obtain solution for a certain value of the heat conductivity is shown in Table 20.2.

Discussion

The benefit of the wave-like equation formulation of the convective transport problem is that this formulation can be discretized without using stabilized finite element formulations. Thus all the subproblems arising from operator splitting can be solved using standard FE techniques. On the other hand, the expense of this approach is that one is lead to handle the second order in time equation.

Table 20.2: CPU time used by the different methods for simulation ($k = 0.01$).

Method	CPU
OS	211.13
S	82.16
RFB	340.70

In the current implementation of the method the wave-like equation is discretized in time using the Crank-Nicolson scheme which is expected to perform well if the solution to the convective transport problem is smooth. Spurious oscillations may however occur in the case of a rough solution changing rapidly in short length scales. The user of the method should be aware that the deterioration of accuracy may thus occur if the solution is not smooth and $\epsilon = k/||\vec{u}||_{\infty} \rightarrow 0$, meaning that convection dominates.

In the cases considered convection dominates, the parameter ϵ ranging between $3.5 \cdot 10^{-2}$ and $7.1 \cdot 10^{-3}$, and the solution has also sharp boundary layer near the upper edge. No spurious oscillations are however detected in the solution. Nevertheless, the results recorded in Table 20.1 indicate that the differences between the methods become larger as $\epsilon \rightarrow 0$.

In view of the results shown in Table 20.2 the accuracy of the operator splitting scheme in predicting the maximum temperature is comparable to that of the residual-free-bubbles method while the computational cost measured in CPU time is reduced by approximately 40 % when the operator splitting scheme is used.

It should be noted that the operator splitting scheme has the favourable feature that small spurious oscillations present in the solution after pure convection step may naturally be damped by the step involving diffusion phenomena. Note also that each of the subproblems arising from the operator splitting may be solved very efficiently using multigrid techniques, whereas robust multigrid solvers amenable to solving linear systems arising from direct discretization of the convection-diffusion equation are still to be found. This makes the operator splitting scheme attractive in the solution of problems having a very large number of unknowns.

Tutorial 21

Temperature distribution with BEM

Directory: PoissonBEM

Solvers: PoissonBEMSolver

Tools: ElmerGrid, editor

Dimensions: 2D

Case definition

This tutorial uses boundary element method (BEM) to solve Poisson equation. Even though Elmer is primarily a finite element software the are limited support also for BEM computation. One should however note that Elmer does not include any multilevel strategies essential for a good performance. For more details about BEM check the Elmer Models Manual. The simulation setting is described in Figure 21.1. A heater with constant heat flux is placed inside a box and the walls of the box are in fixed temperature. We are interested in the temperature distribution in the medium around the heater (Ω) and on the surface of the heater (Γ_1). We also want to know the heat flux through the walls of the box (Γ_2).

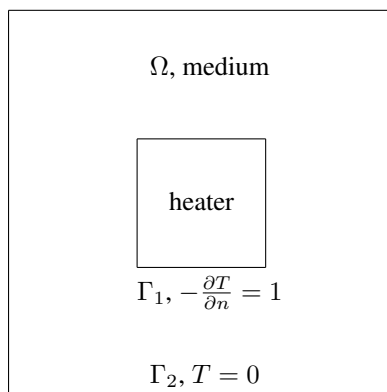


Figure 21.1: Simulation setting

Solution Procedure

First we create a mesh with ElmerGrid. The mesh is defined in `heater.grd` and it is created with command

```
ElmerGrid 1 2 heater
```

The solver input file `PoissonBEM.sif` starts with the definition of the mesh directory.


```
Header
  Mesh DB "." "heater"
End
```

The simulation uses 2D cartesian geometry, searches a steady state and since there is no coupled solvers only one iteration is needed. Numerical results are written to file `BEM_Temperature.result` and ElmerPost file is `BEM_Temperature.ep`.

```
Simulation
  Coordinate System = Cartesian 2D
  Coordinate Mapping(3) = 1 2 3

  Simulation Type = Steady
  Steady State Max Iterations = 1

  Output Intervals = 1
  Post File = "BEM_Temperature.ep"
  Output File = "BEM_Temperature.result"
End
```

There is just one body, the medium around the heater, and it uses equation 1.

```
Body 1
  Name = "medium"
  Equation = 1
End
```

In equation block we say that we use the solver named `PoissonBEM`.

```
Equation 1
  PoissonBEM = Logical True
End
```

In solver block the `Equation` keyword must match the one in equation block. We also need to define the procedure, name the variable (`Temperature`) and tell the degrees of freedom of the variable. Keyword `Optimize Bandwidth` must be set to `false` with BEM solver. Since we were interested in the flux, we must now export it to the results. The lines beginning `Exported` must be exactly as below. Keywords beginning `Linear System` can be used except that the preconditioning cannot be `ILU`.

```
Solver 1
  Equation = PoissonBEM
  Procedure = "PoissonBEM" "PoissonBEMSolver"
  Variable = Temperature
  Variable DOFs = 1

  Optimize Bandwidth = False

  Exported Variable 1 = String Flux
  Exported Variable 1 DOFs = 1

  Linear System Solver = Iterative
  Linear System Iterative Method = BiCGStab
  Linear System Preconditioning = Jacobi
  Linear System Max Iterations = 100
  Linear System Convergence Tolerance = 1.0e-8

  Steady State Convergence Tolerance = 1.0e-6
End
```

Finally we give the boundary conditions for the heater surface and for the walls of the box. The keyword `Body Id` tells the reference body of this boundary. Here it is 1. The keyword `Normal Target Body` tells the direction of the outer normal. Value -1 means the side where there are no volume elements. We didn't mesh the inside of the heater and so we can use value -1 in both cases. The heat flux from heater to medium is 1 and the walls of the box are set to zero temperature. The keyword `Temperature` matches the name of the variable in solver block.

```
Boundary Condition 1
  Name = "heater_surface"
  Target Boundaries = 1

  Body Id = 1
  Normal Target Body = Integer -1
  Flux = Real 1
End
```

```
Boundary Condition 2
  Name = "box_walls"
  Target Boundaries = 2

  Body Id = 1
  Normal Target Body = Integer -1
  Temperature = 0
End
```

Results

Problem is solved with command `Solver`. The results are then viewed with `ElmerPost`. In Figure 21.2 is the temperature distribution.

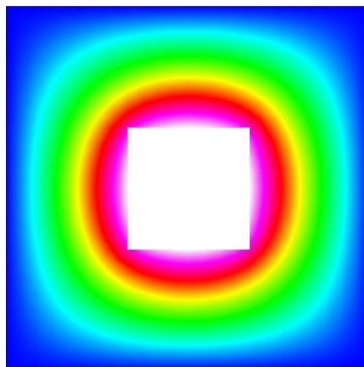


Figure 21.2: The temperature distribution.

Tutorial 22

Adding user defined equation solver

Directory: Temperature1D

Solvers: PoissonSolver

Tools: Editor, Fortran 90 compiler, ElmerGrid

Dimensions: 1D, Steady-state

Problem description

This tutorial is about creating the code for a simple poisson equation solver. The solver is applied to 1d case with internal source term and fixed boundaries.

Mathematically the problem we solve is

$$\begin{cases} -\Delta\Phi = f & \text{in } \Omega \\ \Phi = 0 & \text{on } \Gamma \end{cases} \quad (22.1)$$

Although this example is in 1d the same solver code also applies to 2D and 3D problems.

Solution procedure

Own codes solving some specific equation may be added dynamically to Elmer software. Here we create a very simple equation solver code. The final code may be found in the tutorial directory as well as the files for running the example. The solution may be attempted as follows:

- Copy all the files from tutorial directory to current directory
- Setup Elmer
- Give the following commands:

```
elmerf90 -o Poisson Poisson.f90
ElmerGrid 1 2 1dheat
ElmerSolver
ElmerPost
```

The solver code

The example Fortran code may be found in the tutorial files under the name Poisson.f90. The example run is defined in 1dheat.sif. Only a rough guideline is given here of both of the files, refer to the files themselves for more details.

All the equation solvers in Elmer have the following common interface

```

SUBROUTINE PoissonSolver( Model, Solver, dt, TransientSimulation )
  USE SolverUtils

  TYPE(Model) :: Model
  TYPE(Solver_t), POINTER :: Solver
  REAL(KIND=dp) :: dt
  LOGICAL :: TransientSimulation

  ...
END SUBROUTINE PoissonSolver

```

The argument `Model` contains pointers to the whole definition of the Elmer run. The argument `Solver` contains parameters specific to our equation solver. The argument `dt` and `TransientSimulation` are the current timestep size, and a flag if this run is steady or transient. These don't concern us this time.

When starting the ElmerSolver looks the solver input (.sif) file for a Solver section with keyword "Procedure". This should contain reference to the compiled code

```
Procedure = "Poisson" "PoissonSolver"
```

where the first string in the right hand side is the file name of the compiled code, and second argument is the name of the subroutine to look for in the given file.

In the Solver section one also gives the name of the field variable (here `Poisson`) and the DOFs/node (here 1).

The basic duty of the equation solver is to solve one or more field variables inside the time progressing- or steady state iteration-loop of ElmerSolver. Here we use FEM to discretize the Poisson equation and finally solve the equation by calling ElmerSolver utility `SolveSystem`.

The solution progresses the following way:

- Get the space for variables and temporaries from ElmerSolver and compiler. The matrix structure and space for solution and RHS vector have already been allocated for you before you enter the equation solver.

The matrix is of type `Matrix_t` and may be obtained from the arguments as

```
TYPE(Matrix_t), POINTER :: StiffMatrix
StiffMatrix => Solver % Matrix
```

Usually one doesn't need to know the internal storage scheme or the fields of the `Matrix` type, but one just passes this pointer further to ElmerSolver utility routines.

Similarly, the force vector may be accessed as follows:

```
REAL(KIND=dp), POINTER :: ForceVector(:)
ForceVector => StiffMatrix % RHS
```

The solution vector is obtainable similarly

```
TYPE(Variable_t), POINTER :: Solution
Solution => Solver % Variable
```

The `Variable_t` structure contains the following fields

- DOFs: the number of degrees of freedom for one node. This value is for information only and should'nt be modified.
- Perm: an integer array that is nonzero for nodes that belong to computational volume for this equation. The entry $Perm(i)$ holds the index of the global matrix row (for 1 DOF) for nodal point i . This array should'nt be modified by the equation solver.

- Values: Space for the solution vector values. Note that the values are ordered the same way as the matrix rows, i.e. the value of Potential at node n is stored at

```
val = Solution % Values( Solution % Perm(n) )
```

- Initialize the global system to zero. Calling the utility routine

```
CALL InitializeToZero( StiffMatrix, ForceVector )
```

is usually enough.

- Go through the elements for which this equation is to be solved, get the elemental matrices and vectors and add them to the global system:

```
DO i=1, Solver % NumberOfActiveElements
  CurrentElement => Solver % Mesh % Elements( Solver % ActiveElements(i) )
  ...
  CALL LocalMatrix( ... )
  CALL UpdateGlobalEquations( ... )
END DO
CALL FinishAssembly( ... )
```

Here the LocalMatrix is your own subroutine computing elemental matrices and vectors. In the example code LocalMatrix uses three routines from ElmerSolver utilities. The function

```
dim = CoordinateSystemDimension()
```

returns the dimension of the current coordinate system, i.e. the return value is 1, 2 or 3 depending on the input file setting of keyword "Coordinate System". The function GaussPoints returns structure containing the integration point local coordinates and weights

```
TYPE(GaussIntegrationPoints_t) :: IntegStuff
IntegStuff = GaussPoints( Element )
```

The fields of the type GaussIntegrationPoints_t are

```
INTEGER :: n
REAL(KIND=dp) :: u(:), v(:), w(:), s(:)
```

the integer value n is the number of points selected. The arrays u,v and w are the local coordinates of the points, and the array s contains the weights of the points. One may call the GaussPoints-routine with second argument,

```
IntegStuff = GaussPoints( Element, n )
```

if the default number of integration points for given element is not suitable.

Inside the integration loop the function ElementInfo is called:

```
TYPE(Element_t), POINTER :: Element
TYPE(Nodes_t) :: Nodes
REAL(KIND=dp) :: U,V,W,detJ, Basis(n), dBasisdx(n,3), ddBasisddx(n,3,3)

stat = ElementInfo( Element, Nodes, U, V, W, detJ, &
  Basis, dBasisdx, ddBasisddx, .FALSE. )
```

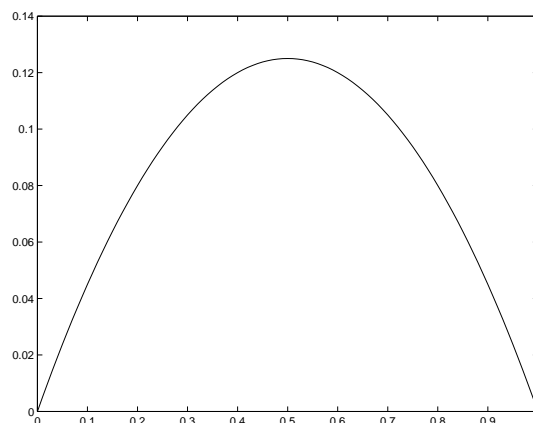


Figure 22.1: Solution of the Poisson Equation.

This routine returns determinant of the element jacobian ($\det J$), basis function values ($\text{Basis}(n)$), basis function global derivative values ($d\text{Basis}dx(n,3)$), basis function second derivative values ($dd\text{Basis}ddx(n,3,3)$). The second derivatives are only computed if the next logical flag is set to true. All the values are computed at the point U, V, W inside element defined by structures Element and Nodes .

Refer to the code for more details.

- Set boundary conditions. Here only dirichlet boundary conditions are used. These may be set by using the utility routine `SetDirichletBoundaries`.
- Solve the system by calling utility routine `SolveSystem`.

Results

In the `elmerpost` file there is a variable called `Potential` which contains the solution of this simple example. See figure 22.1

Tutorial 23

Volume flow boundary condition

Directory: FlowLinearRestriction

Solvers: FlowSolve, SolveWithLinearRestriction

Tools: Editor, Fortran 90 compiler, ElmerGrid

Dimensions: 2D, Transient

Case definition

This tutorial gives an example how to use SolveWithLinearRestriction. It also describes how to execute own functions before the original system is solved. In order to understand the case reader should be familiar with compressed row storage matrixes and elmer basics. This tutorial gives only the guidelines and reader is advised to read the files in order to get more through understanding.

We simulate the flow of incompressible fluid in a pipe. The pipe has a length of 5 and a width of 1. On the left end we want to describe a certain time dependent volume flow. In other words, we don't want to describe the velocity field here but we want the velocity field be such that it transports certain amount of volume in time interval. We could integrate the correct volume flow, but let's now approximate it to make the more important aspects more visible. Our approximation here is that the volume flow is proportional to average velocity on the edge i.e.

$$\frac{1}{N} \sum_{i=1}^N u_i = \frac{volume}{time} \quad (23.1)$$

Here u_i are the nodal velocities parallel to the pipe on the left edge and N is the number of nodes on the left edge. We want to set a nicely scaled sinusoidal volume flow on the edge, which leads to

$$\sum_{i=1}^N u_i = 10N \sin(2\pi t) \quad (23.2)$$

This equation we can (easily) force with lagrange multiplier.

Solution procedure

First we make a uniform mesh of 800 four-node quadrilaterals with command

```
ElmerGrid 1 2 mflow
```

Next we construct the solver input file. Header is simply

```
Header
  Mesh DB "." "mflow"
End
```

The simulation block is also very simple. Here we need to define the timestepping method and timescale.

```

Simulation
  Coordinate System = Cartesian 2D

  Simulation Type = Transient
  Steady State Max Iterations = 1

  Timestepping Method = BDF
  BDF Order = 1

  Timestep Sizes = 0.02
  Timestep Intervals = 100

  Output Intervals = 1

  Output File = "mflow.result"
  Post File = "mflow.ep"
End

```

The body, material and equation blocks are as usual. The material parameters, of course, have affect on the solution and interested reader is encouraged to modify these values and recalculate the solution.

```

Body 1
  Material = 1
  Equation = 1
End

Material 1
  Density = 3.0
  Viscosity = 0.1
End

```

```

Equation 1
  Navier-Stokes = TRUE
  Active Solvers(1) = 1
End

```

The solver block has the usual Navier-Stokes keywords and two keywords for volume flow boundary. The `Before Linsolve` keyword defines binaryfile and function that is called before the system is solved. This function we must write and compile and we will come to it shortly. The following keyword, `Export Lagrange Multiplier`, states that we are not interested in the value of the Lagrange multiplier and it is therefore not saved.

```

Solver 1
  Equation = Navier-Stokes
  Stabilize = True

  Before Linsolve = "./AddMassFlow" "AddMassFlow"
  Export Lagrange Multiplier = Logical FALSE

  Linear System Solver = Iterative
  Linear System Iterative Method = BiCGStab
  Linear System Preconditioning = ILU1
  Linear System Max Iterations = 500
  Linear System Scaling = False
  Linear System Convergence Tolerance = 1.0e-8

```



```

Nonlinear System Max Iterations = 15
Nonlinear System Convergence Tolerance = 1.0e-8
Nonlinear System Newton After Tolerance = 1.0e-4
Nonlinear System Newton After Iterations = 8
Nonlinear System Relaxation Factor = 1.0

```

```

Steady State Convergence Tolerance = 1.0e-7
End

```

In boundary conditions we state that both parallel and perpendicular velocities are zero on the pipe sides and on both edges the perpendicular velocity is zero. Here we also define the number tags for the boundaries. The tag 2 is assigned to boundary that has number 4 in `grd`-file, which is the left edge of the pipe. To this tag number 2 we shall refer in our `AddMassFlow`-function.

```

Boundary Condition 1
  Target Boundaries(2) = 1 3
  Velocity 1 = 0.0
  Velocity 2 = 0.0
End

```

```

Boundary Condition 2
  Target Boundaries = 4
  Velocity 2 = 0.0
End

```

```

Boundary Condition 3
  Target Boundaries = 2
  Velocity 2 = 0.0
End

```

AddMassFlow function

Here we shall only give some rough guidelines of the function, for more information check the code. This function creates the constraint matrix and RHS that forces the equation mentioned above. Then it calls `SolveWithLinearRestriction` to solve the system. The coinstraint matrix is actually only a row-vector and the RHS is only one value.

- The function parameters are defined in Elmer so you shouldn't change them.
- First we set a pointer to `EMatrix`-field of the given system matrix. If the pointed matrix is not yet allocated, calculate the number of nodes on the edge we want to define the volume flow. This gives us the number of non-zeros in our constraint matrix and we can allocate the matrix.
- Set the rows, cols and diag -fields of the matrix. This sets the non-zeros on their right places in the constraint matrix.
- Set all values of the constraint matrix to unity.
- Calculate the RHS-value. The current time was checked in the beginning of the function, so this is possible.
- Call `SolveWithLinearRestriction`
- Return 1 which tells the `ElmerSolver` that the system is already solved.

The function is compiled with command

```
elmerf90 -o AddMassFlow AddMassFlow.f90
```

Here it is assumed that the source file name is `AddMassFlow.f90`.

Results

Just say `ElmerSolver` and you should get the solution in few minutes. The velocity perpendicular to the pipe is practically zero and the velocity parallel to the pipe is an example of Womersley velocity profile ¹. An interesting feature of this velocity profile is that on some timesteps the fluid flows to both directions, see figure 23.1.

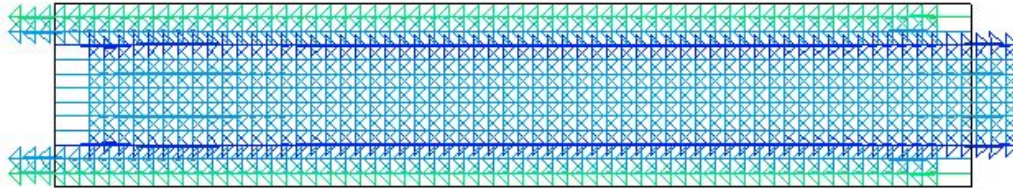


Figure 23.1: Solution of the velocity field. Note the flow to both directions.

¹J.Physiol (1955) 127, 553-563

Tutorial 24

Streamlines

Directory: FlowStreamlines
Solvers: StreamSolver, FlowSolve
Tools: ElmerGrid, editor
Dimensions: 2D

Case definition

The case definition is the same as in the incompressible flow passing a step. The mathematical definition of the stream function ψ is

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (24.1)$$

where u, v are the velocity components in x, y geometry. For more info check Elmer Models Manual.

Solution Procedure

First we create a mesh with ElmerGrid. The mesh is defined in `step.grd` and it is created with command

```
ElmerGrid 1 2 step
```

You may need to compile the StreamSolver yourself. If the Elmer environment is successfully setup the compilation command should look like the following lines,

```
elmerf90 -o StreamSolver StreamSolver.f90
```

The solver input file `streamlines.sif` starts with the definition of the mesh directory.

```
Header
  Mesh DB "." "step"
End
```

The simulation uses 2D cartesian geometry and searches a Steady State. There is no coupled solvers so only one iteration is needed. Numerical results are written to file `streamlines.result` and ElmerPost file is `streamlines.ep`.

```
Simulation
  Coordinate System = Cartesian 2D
  Coordinate Mapping(3) = 1 2 3

  Simulation Type = Steady
  Steady State Max Iterations = 1
```

```

Output Intervals = 1
Post File = "streamlines.ep"
Output File = "streamlines.result"
End

```

There is just one body and it uses equation 1 and is of material 1.

```

Body 1
Equation = 1
Material = 1
End

```

The equation block states that we use Solvers 1 and 2 to solve the problem and that we use Navier-Stokes equations.

```

Equation 1
Active Solvers(2) = 1 2
Navier-Stokes = True
End

```

In material block we define the density and the viscosity of the fluid.

```

Material 1
Density = 1
Viscosity = 0.01
End

```

Solver 1 is for the Navier-Stokes equations. Here we give the linear system solver ¹ and convergence criteria for linear, nonlinear and steady state solution of the Navier-Stokes equations.

```

Solver 1
Equation = "Navier-Stokes"
Stabilize = True

Linear System Solver = Iterative
Linear System Iterative Method = BiCGStab
Linear System Max Iterations = 500
Linear System Convergence Tolerance = 1.0e-8
Linear System Preconditioning = ILU1

Nonlinear System Convergence Tolerance = 1.0e-6
Nonlinear System Max Iterations = 15
Nonlinear System Newton After Iterations = 8
Nonlinear System Newton After Tolerance = 1.0e-4
Nonlinear System Relaxation Factor = 1.0

Steady State Convergence Tolerance = 1.0e-6
End

```

Then the solver for streamlines.

- Name of the equation. This may be what ever you like.
- Name of the binary file and the subroutine. If you compiled the StreamSolver yourself, then you may need to change this to Procedure = "./StreamSolver" "StreamSolver".
- Name of the variable. This may be what ever you like.

¹Biconjugate gradient method with incomplete LU preconditioning

- Stream function is scalar, so the degree of freedom is 1.

Next set of keywords is for the StreamSolver. More info on keywords is in the Elmer Models Manual.

- Name of the flow field variable. The name of the FlowSolves variable is FlowSolution.
- Global number of the offset node. 1 is always a safe choice.
- Shift the smallest value to zero.
- Scale the maximum value to 1.
- Use the normal stream function i.e. don't use Stokes stream function.

Then we define the linear system solver and convergence criterions.

```
Solver 2
Equation = "StreamSolver"
Procedure = "StreamSolver" "StreamSolver"
Variable = "StreamFunction"
Variable DOFs = 1

Stream Function Velocity Variable = String "Flow Solution"
Stream Function First Node = Integer 1
Stream Function Shifting = Logical TRUE
Stream Function Scaling = Logical TRUE
Stokes Stream Function = Logical FALSE

Linear System Solver = Iterative
Linear System Iterative Method = BiCGStab
Linear System Max Iterations = 500
Linear System Convergence Tolerance = 1.0e-8
Linear System Preconditioning = ILU1

Steady State Convergence Tolerance = 1.0e-6
End
```

Finally we give the boundary conditions. The condition 1 is for the lower and upper side of the step ($\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_5$ in case definition). Here both velocities are zero. The condition 2 is for the output edge (Γ_4). Here vertical velocity is zero. The condition 3 is for the input edge (Γ_6). Here horizontal velocity is 1 and vertical velocity is zero.

```
Boundary Condition 1
Target Boundaries = 1
Velocity 1 = 0
Velocity 2 = 0
End
```

```
Boundary Condition 2
Target Boundaries = 2
Velocity 2 = 0
End
```

```
Boundary Condition 3
Target Boundaries = 3
Velocity 1 = 1
Velocity 2 = 0
End
```

Results

Problem is solved with command `Solver`. The results are then viewed with ElmerPost. In figure 24.1 are some contour lines of the stream function. These are also flows streamlines. The contour values are manually selected to get a nice picture. Note the swirl after the step.

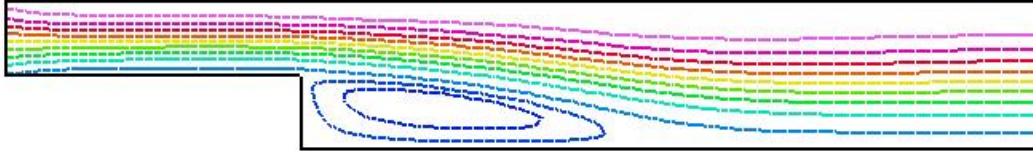


Figure 24.1: The streamlines of the flow.

Tutorial 25

Electroosmotic flow and advected species

Directory: Microfluidic

Solvers: StatElecSolve, FlowSolve , AdvectionDiffusion, Electrokinetics

Tools: ElmerGrid, editor

Dimensions: 2D

Case definition

This tutorial is an example of setting up a simulation for (microfluidic) electroosmotic flow advecting a passive scalar quantity. Diffusion of the species is also included. The geometry of the system is a simple 2D microchannel with T crossing. The flow is induced by the applied electric field and the electric double layer at the channel walls. The analyte (species) is inserted into the system from the left hand side inlet.

More details on the electrokinetic capabilities of Elmer are found on the Models Manual, chapter “Electrokinetics”.

Solution Procedure

The computational mesh is done with ElmerGrid in directory Tcross with the command

```
ElmerGrid 1 2 Tcross -scale 1e-5 1e-5 1e-5
```

The scale option above is used to obtain appropriate dimensions from a geometry template defined in non-dimensional units.

The command file may be written with a text editor. The file includes the following information.

The mesh directory is given in the header of the command file

```
Header
  Mesh DB "." "Tcross"
End
```

The simulation block of the command file defines, eg., the case to be time dependent (transient) with 10^{-5} second timesteps and altogether 120 time intervals. Results from every second timestep are saved into the files diffusion1.*.

```
Simulation
  Coordinate System = Cartesian 2D

  Simulation Type = "Transient"
  Steady State Max Iterations = 20
```

```

Timestep Intervals = 120
Timestep Sizes = 1e-5
Output Intervals = 2

Timestepping Method = BDF
BDF Order = 2

Binary Output = Logical True

Output File = "diffusion1.res"
Post File = "diffusion1.ep"

Output Version Numbers = Logical True
Max Output Level = 32
End

```

The electrostatics and electrokinetics solvers require the value of the permittivity of vacuum. This is actually not even needed here since the case deals with conducting media. Thus the value has been fixed as 1.0 to avoid warnings on missing constant definitions.

```

Constants
! Permittivity Of Vacuum = 8.8542e-12 ! C^2/Nm^2
  Permittivity Of Vacuum = 1.0 ! manipulation for conducting material
End

```

The case includes only one body. The corresponding equation definitions are found in section Equation 1 and material parameters from section Material 1.

```

Body 1
  Equation = 1
  Material = 1
End

```

All three solvers are active in this equation set. Further definitions include, first, that the convection of the species is switched on (besides diffusion). Then that for Navier-Stokes equations the convective term may be left out resulting in laminar Stokes flow, and finally that the electric field is computed by the electrostatics rather than given by the user.

```

Equation 1
  Active Solvers(3) = 1 2 3
  Convection = Computed
  NS Convect = False
  Electric Field = String "computed"
End

```

Following are the solver definitions. Solver 1 is the electrostatics solver. The equation is linear and thus no nonlinear iterations are needed. The equation is solved using a fast direct method UMFPack.

```

Solver 1
  Equation = "Stat Elec Solver"
  Procedure = "StatElecSolve" "StatElecSolver"
  Variable = String "Potential"
  Variable DOFs = 1

  Calculate Electric Field = True
  Calculate Electric Flux = False
  Linear System Convergence Tolerance = 1.0E-10

```



```

Linear System Solver = Direct
Linear System Direct Method = UMFPack
Linear System Preconditioning = ILU1
Linear System Residual Output = 1

Nonlinear System Max Iterations = 1
Nonlinear System Convergence Tolerance = 1.0e-10
Steady State Convergence Tolerance = 1.0E-10
End

```

The next solver is for the Navier-Stokes equations. Here nonlinear iterations are required.

```

Solver 2
Equation = "Navier-Stokes"

Linear System Convergence Tolerance = 1.0D-08
Linear System Solver = Iterative
Linear System Iterative Method = "BiCGStab"
Linear System Max Iterations = 500
Linear System Abort Not Converged = False ! True
Linear System Preconditioning = ILU1
Linear System Residual Output = 10

Nonlinear System Convergence Tolerance = 1.0e-6
Nonlinear System Max Iterations = 30
Nonlinear System Newton After Iterations = 10
Nonlinear System Newton After Tolerance = Real 1.0D-8
Nonlinear System Relaxation Factor = 1.0
Steady State Convergence Tolerance = 1.0D-03

Stabilize = True
End

```

The advection-diffusion equation does not affect neither the electrostatic field nor the flow, thus it may be solved only after a converged solution for the previous two equations is available. This is achieved with the `Exec Solver` definition below. The advected quantity is given the name `Analyte`. The advection-diffusion solver uses bubble stabilization method to avoid numerical problems associated with convection type equations.

```

Solver 3
Exec Solver = After Timestep
Equation = "Analyte transfer"
Procedure = "AdvectionDiffusion" "AdvectionDiffusionSolver"
Variable = String "Analyte"
Variable DOFs = 1

Bubbles = True

Linear System Convergence Tolerance = 1.0E-06
Linear System Solver = "Iterative"
Linear System Iterative Method = "BiCGStab"
Linear System Max Iterations = 500
Linear System Preconditioning = ILU2
Linear System Residual Output = 1

```

```

Nonlinear System Max Iterations = 1
Nonlinear System Convergence Tolerance = 1.0e-5
Steady State Convergence Tolerance = 1.0D-06
End

```

The material parameters are given below.

```

Material 1
  Density = 1e3

  Viscosity = 1e-03

  Relative Permittivity = 1.0    ! this is actually electric conductivity

  Analyte Diffusivity = Real 1e-10
End

```

Finally the boundary conditions are defined. The first BC is given for the channel walls. Here, tangential velocity (velocity components 1 and 2) is computed by the Helmholtz-Smoluchowski slip velocity condition, which means that the velocity is computed using the computed electric field and the electroosmotic mobility as inputs.

```

Boundary Condition 1
  Name = "channel-walls"
  Target Boundaries(2) = 4 5

  EO Mobility = Real 5e-08

  Velocity 1 = Variable Pressure
    Real Procedure "Electrokinetics" "helmholtz_smoluchowski1"
  Velocity 2 = Variable Pressure
    Real Procedure "Electrokinetics" "helmholtz_smoluchowski2"
End

```

The next BC is the inlet condition. We give a potential of 100 Volts and define that there is no flow in y -direction. The analyte concentration at the inlet is defined as a function of time using table format. The concentration is 1.0 up until time instant $3 \cdot 10^{-5}$, is zero after $4 \cdot 10^{-5}$ s and decreases linearly between these two time instants.

```

Boundary Condition 2
  Name = "el_A"
  Target Boundaries = 1

  Potential = 100.0

  Velocity 2 = 0.0

  Analyte = Variable Time
    Real
      0.0      1.0
      3.0e-5   1.0
      4.0e-5   0.0
      0.5      0.0
    End
End

```

The final two boundary conditions are for the outlets. Different potentials for these are defined as well as a condition for velocity component.

```
Boundary Condition 3
  Name = "el_B"
  Target Boundaries = 2

  Potential = 30.0

  Velocity 1 = 0.0
End
```

```
Boundary Condition 4
  Name = "el_C"
  Target Boundaries = 3

  Potential = 0.0

  Velocity 1 = 0.0
End
```

After writing the command file is finished, the problem can be solved by entering the command `ElmerSolver` on the command line. The results can be examined with `ElmerPost`.

Results

Solving the problems takes less than a minute cpu time on a PC. The maximum and minimum concentration over the whole simulation are 1.0235 and -0.075748. The solution of this problem should be between 0 and 1. This shows that some numerical discretization errors are present in the simulation. The errors would diminish when using smaller timesteps and also with denser mesh. Simulation results at the time instant of 0.00025 seconds are shown in Fig. 25.1.

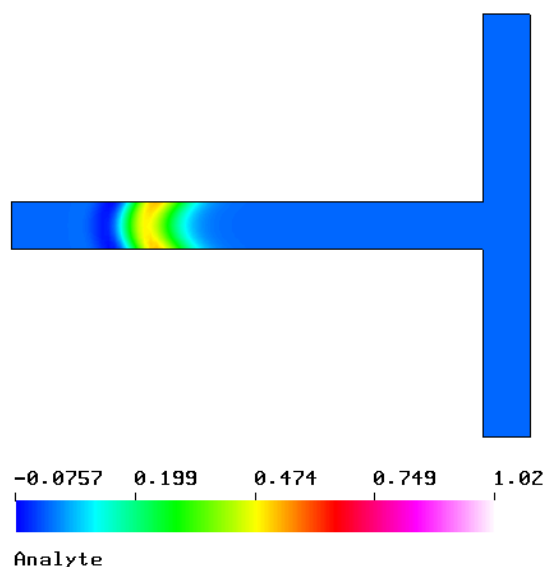


Figure 25.1: Analyte concentration on time instant 0.00025 seconds.

The maximum value of the magnitude of the velocity in the results is 0.105 m/s.

The electric field is written into the output only componentwise. The magnitude of the field may be visualised after giving the following commands on the ElmerPost command line (assuming one has read in all 61 timesteps):

```
math E(0,time(0):time(61)-1) = Electric.field.1
math E(1,time(0):time(61)-1) = Electric.field.2
math E(2,time(0):time(61)-1) = 0
math E_abs = sqrt(vdot(E,E))
```

Now visualising the variable `E_abs` reveals that the electric field magnitude is between 2453 and $2.18 \cdot 10^6$.

Notes

When checking the simulation results the user will notice that the electric potential does not change in time and that the flow reaches steady-state within a few timesteps. This is quite clear also from the problem setup: the electric field is due to invaring potential with constant material parameters. Also the flow in microchannels is usually laminar.

Thus the most efficient way to simulate the current case would be to compute first the steady-state solution for the electrokinetic flow and use the steady flow to advect the analyte. This would be done by running two separate simulations; resolve first the steady flow, and to use this flow solution as restart for the advection-diffusion equation.

Tutorial 26

Active and passive elements

Directory: PassiveElements
Solvers: HeatSolve
Tools: ElmerGrid, editor
Dimensions: 2D

Case definition

This tutorial shows an example of using passive elements in Elmer. This feature allows the activation and deactivation of parts of the geometry during the simulation. This tutorial uses the heat equation solver to demonstrate this capability. Use with other solvers is analogous.

The geometry of the problem consists of two parts. The lower edge of the lower part is held at constant temperature of 1 degrees. The upper body is heated with a constant heating power. Between time steps 5 and 6 the two bodies are connected by two heat conductors, and the heat is conducted from the higher body to the lower one. The goal of the simulation is to model the temperature distribution in the system over time.

The problem is a pure heat transfer problem that may be solved with `HeatSolve`.

Solution Procedure

The computational mesh is done with `ElmerGrid` in directory `tmesh` with the command

```
ElmerGrid 1 2 tmesh
```

The command file may be written with a text editor. The file includes the following information.

The mesh directory is given in the header of the command file

```
Header
  Mesh DB "." "tmesh"
End
```

The simulation block of the command file defines, eg., the case to be time dependent (transient) with 1 second timesteps and altogether 15 time intervals.

```
Simulation
  Max Output Level = 32
  Coordinate System = Cartesian 2D
  Simulation Type = Transient
  Timestepping Method = BDF
  BDF Order = 2
  Timestep Intervals = 15
  Timestep Sizes = 1
  Output Intervals = 1
```

```

Steady State Max Iterations = 1
Output Version Numbers = Logical True
Output File = heat.res
Post File = heat.ep
End

```

The heat equation solver asks for the Stefan-Boltzmann constant that gives the relationship between temperature and radiation power, although radiation is not needed here. Let us define it anyway to avoid warnings of missing parameters.

```

Constants
  Stefan Boltzmann = 5.67e-8
End

```

There are three bodies with the same equation but different material properties. Body 3 is heated by a constant body force. Body 2 forms the connecting parts of the system. An initial condition as well as a body force is defined for this body. The body force contains the initial deactivation, and later activation, of the connecting part. Note that this part is included in the geometry all the time, but the command file is used to define when they are included into the simulation.

```

Body 1
  Equation = 1
  Material = 1
End

Body 2
  Equation = 1
  Material = 2
  Body Force = 2
  Initial Condition = 1
End

Body 3
  Equation = 1
  Material = 1
  Body Force = 1
End

```

The only solver is the heat solver (Solver 1)

```

Equation 1
  Active Solvers = 1
End

```

The initial condition for the initially passive elements is taken to be 1 degrees; the same temperature than the colder part of the system has as a boundary condition.

```

Initial Condition 1
  Temperature = 1.0
End

```

The heating power is defined to be 10 W/kg

```

Body Force 1
  Heat Source = 10
End

```

Now the passive condition for the connecting part is defined. When the parameter `Temperature Passive` has a value larger than zero, the current element is excluded from the solution, otherwise it is included as a normal element. The parameter may depend on variables, coordinates or time. Here it is defined to depend on time using a tabular format.

```

Body Force 2
  Temperature Passive = Variable Time
  Real
    0.0    1.0
    5.0    1.0
    5.2   -1.0
    8.0   -1.0
  End

```

End

The material properties of the system are artificial. The following three properties are needed for each material.

```

Material 1
  Heat Capacity = 1
  Heat Conductivity = 1
  Density = 1
End

```

```

Material 2
  Heat Capacity = 10
  Heat Conductivity = 1
  Density = 1
End

```

The heat equation is solved with an iterative method. The system is linear, thus multiple nonlinear iterations are not needed.

```

Solver 1
  Equation = heat equation
  Linear System Solver = Iterative
  Linear System Iterative Method = BiCGStab
  Linear System Preconditioning = ILU0
  Linear System Max Iterations = 300
  Linear System Convergence Tolerance = 1.0e-6
  Linear System Abort Not Converged = Logical False
  Nonlinear System Max Iterations = 1
  Nonlinear System Convergence Tolerance = 1.0e-5
  Steady State Convergence Tolerance = 1.0e-5
End

```

The boundary conditions are simple. The lower boundary of the lower body is held at 1 degrees and the upper boundary of the upper body at 10 degrees.

```

Boundary Condition 1
  Target Boundaries = 1

  Temperature = 1
End

```

```

Boundary Condition 2
  Target Boundaries = 4

  Temperature = 10
End

```

After writing the command file is finished, the problem can be solved by entering the command `ElmerSolver` on the command line. The results can be examined with `ElmerPost`.

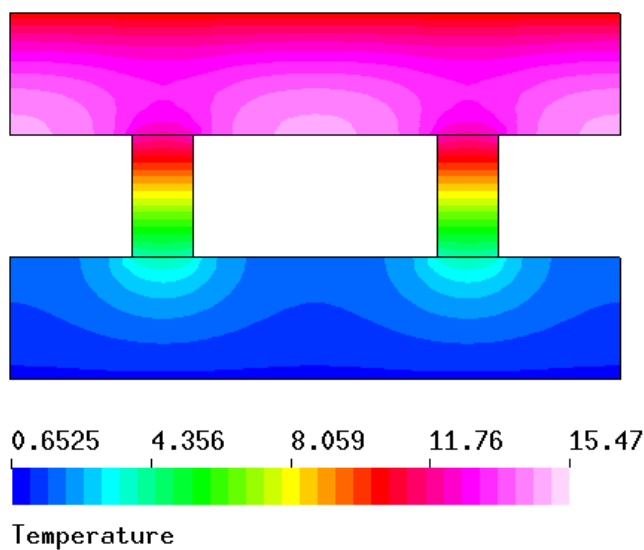


Figure 26.1: Temperature distribution of the system at the final time instant (with spectral_32 color map).

Results

With the given computational mesh the problem is solved in a few seconds. The maximum and minimum temperatures in the system over the whole simulation are 15.466 degrees and 0.6525 degrees respectively. The maximum and minimum temperature at the final time instant are 14.207 degrees and 1.000 degrees, respectively. The results at the time instant of 15 seconds are shown in Fig. 26.1.

Notes

For equations with more than one components (such as displacement for Stress Analysis solver in 2D or 3D) the passive elements feature apply to all the components. The feature is activated by defining, eg., `Displacement Passive` in the `Body Force` section. Note that for Navier-Stokes equations one should use `Flow Solution Passive`, and that this affects the Pressure as well as the Velocity components.

However, when using multiple solvers, one can define some of them passive and some of them active at the same time.

Part IV

Obsolite Problems

These tutorials have become obsolete due to the transition from ElmerFront to ElmerGUI. If you still want to continue using ElmerFront these tutorials may be perfectly valid.

Tutorial 27

Temperature distribution

Directory: TemperatureAngle
Solvers: HeatSolve
Tools: ElmerFront
Dimensions: 2D, Steady-state

Problem description

An L-shaped structure (see figure 27.1) is heated by an internal heat source, which magnitude is 1 W/m^3 . The density of the structure is 1 kg/m^3 and the heat conductivity is 1 W/mK . All the boundaries Γ_i are kept on constant temperature of 0 K . The problem is to solve the temperature distribution in the structure.

Mathematically the problem to be solved is

$$\begin{cases} -\kappa\Delta T & = & \rho f & \text{in}\Omega \\ T & = & 0 & \text{on}\Gamma \end{cases} \quad (27.1)$$

where κ is the heat conductivity, T is the temperature and f is the heat source. It is assumed that density and heat conductivity are constants.

Solution procedure

- Start Elmer in the desired directory with the command

```
ElmerFront
```

- Open the file that contains the geometry of the structure from the File menu. Select also the working directory for the model.

```
File -> Open cad-file  
File = TempDist.egf  
Model name = TempDist  
Model directory = temp_tutorial
```

- Select the equations to be solved from the Problem menu. In this case there is only one equation, the heat equation.

```
Problem -> Equations  
Heat equation
```

- Apply the body forces from the Model menu. Give the value for the body force (heat source).

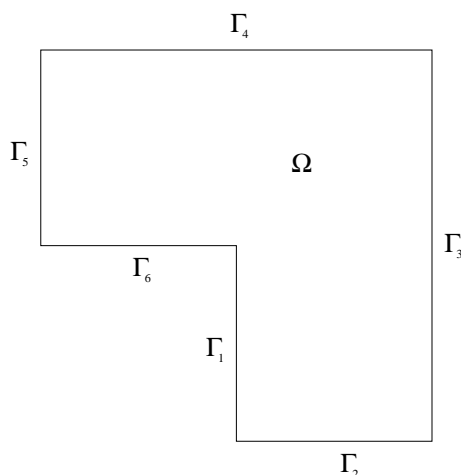


Figure 27.1: L-shaped structure

```
Model -> Body forces
Heat source = 1
```

- Define the material properties from the Model menu. Give the values for the density and the heat conductivity.

```
Model -> Materials
Density = 1
Heat conductivity = 1
```

- Define boundary conditions from the Model menu. Give the value of the temperature at all boundaries $\Gamma_i (i = 1, \dots, 6)$.

```
Model -> Boundary conditions
Temperature = 0
```

- Define mesh for the structure from the Mesh menu. First give name for the mesh and then define the element size. Press finally the “Generate Mesh” button.

```
Mesh -> Define mesh
Mesh name = Mesh1
Model Mesh H [m] = 0.1
Generate mesh
```

- Now the problem may be solved.

```
Run -> Solver
```

- After the solution is done, view the results by selecting the Postprocessor from the Run menu.

```
Run -> Postprocessor
```

- To save the created model, select Save model file from the File menu.

```
File -> Save model file
```

Table 27.1: Results with different element sizes

Model Mesh H [m]	Elements	$\max(T)$ [K]	cpu time [s]
0.2	187	0.1452	0.25
0.15	330	0.1462	0.28
0.1	735	0.1478	0.38
0.05	2793	0.1487	0.85
0.025	11100	0.1492	2.88
0.01	69654	0.1493	19.37

Results

As a result the maximum temperature in the structure is given. For a comparison the same problem was solved six times with different element sizes. The maximum temperature obtained by using different meshes is recorded in Table 27.1. From the results one can see that the result converges. With a denser mesh the result is more accurate, but solving the problem takes more calculation time. For reference, the central processor (cpu) time used in each case is also shown in the table.

Tutorial 28

Loaded elastic beam

Directory: ElasticBeam

28.1 Solution with linear model

Solvers: StressSolve

Tools: ElmerFront

Dimensions: 2D, Steady-state

Case definition

A homogenous, elastic beam (Ω) is rigidly supported on one end (boundary Γ_4). On boundary Γ_3 the beam is subjected to a load $q(x)$, which grows linearly from zero to q_0 (see figure 28.1). Material properties of the beam are the Poisson ratio 0.3 and Young's modulus $200 \cdot 10^9 \text{N/m}^2$. Problem is to solve the displacement of the beam.

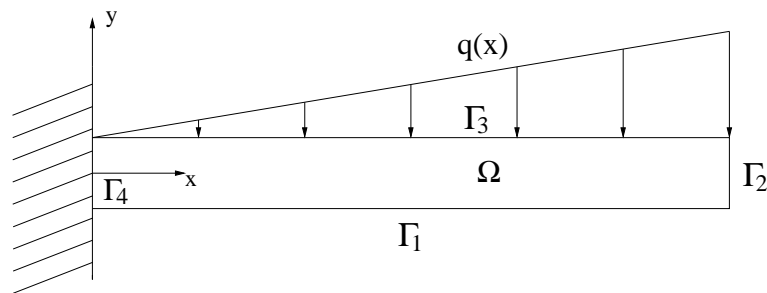


Figure 28.1: Beam and loading.

Problem is solved according to linear elasticity theory. Mathematically the problem to be solved is

$$\left\{ \begin{array}{ll} -div\sigma = 0 & \text{in } \Omega \\ \sigma = \lambda tr[\varepsilon(u)]I + 2\mu\varepsilon(u) & \text{in } \Omega \\ u = 0 & \text{on } \Gamma_4 \\ \sigma n = 0 & \text{on } \Gamma_1 \cup \Gamma_2 \\ \sigma n = -q & \text{on } \Gamma_3 \end{array} \right. \quad (28.1)$$

where λ and μ are the Lamé constants (which can be expressed in terms of the Poisson ratio and Young's modulus), ε is the linearized strain tensor, u is the displacement vector, q is the given surface traction and n is the outward unit normal to the boundary.

Solution procedure

- Start ElmerFront.
- Open the file that contains the geometry of the beam from the File menu. Select also the working directory for the model.

```
File -> Open cad-file
File = Beam.egf
Model name = Beam
Model directory = beam_tutorial
```

- Select the equations to be solved from the Problem menu. In this case stress analysis is selected. It solves the problem according to linear elastic theory.

```
Problem -> Equations
Stress analysis
```

- Define the material properties from the Model menu. Give the values for Young's modulus and the Poisson ratio. Add the defined material properties to the material property sets so they become attached to Ω .

```
Model -> Materials
Young's modulus = 200e9
Poisson ratio = 0.3
```

- Define the Dirichlet boundary condition and the load from the Model menu. Give the value zero for displacements at the boundary Γ_4 and press Add. The linearly varying load is defined in the same panel as follows. Select the boundary 3, click the cursor on to the Force-y line, check the box Table, and press finally the Edit button. A window opens in which the tabular bc entry is defined. Select first Coordinate 1 as the variable on which the Force-y depends. Write on the line below entry 0 0. Click Add. Write 1 -1.0e7 on the line and click again Add. Now, the space below contains two lines written in two columns. The first column holds the values for the Coordinate 1 and the second column for the Force-y. The value of the force is interpolated according to these definitions. Now click OK on the Table entry panel. In Boundary Conditions panel, click Add and then OK.

```
Model -> Boundary conditions
Boundary 4
Displacement-X = 0
Displacement-Y = 0
Add
Boundary 3
Force-Y
Table
Edit
Variable = Coordinate 1
0 0
Add
1 -1e7
Add
OK
Add
OK
```

- Define mesh from the Mesh menu. First give name for the mesh. Then select "Mesh structure" and define element type and the number of elements. Attach the defined mesh structure to the Body 1 and click OK. Create the mesh by pressing "Generate mesh" button.

```
Mesh -> Define mesh
Mesh name = Mesh1
Mesh structure
Element type = Quad
Nof elements (1st and 3rd edge) = 40
Nof elements (2nd and 4th edge) = 4
Add
OK
Generate mesh
```

- Now to solve the problem defined with the constant load select from the Run menu item Solver.

```
Run -> Solver
```

- Results may be viewed with the ElmerPost program

```
Run -> Postprocessor
```

or click the Results button on the main window.

Results

As a result the absolute value of maximum displacement is given. The displacements calculated with different load values q_0 are tabulated in table 28.1. Note that the absolute value of the displacement varies linearly with respect to the load since the model is linear.

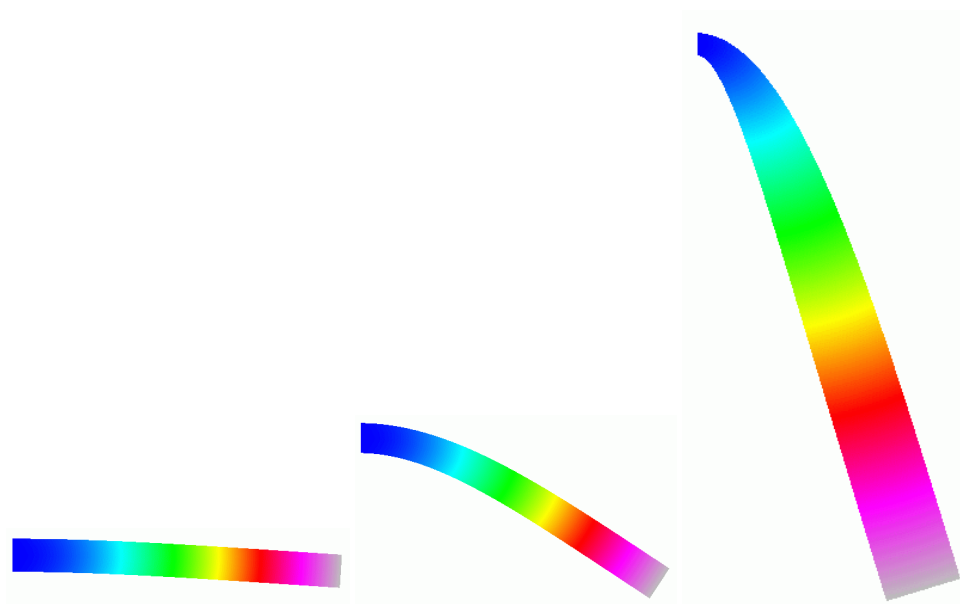


Figure 28.2: The displacement of an elastic beam with different loads using a linear model

If you look at the results you can see that the displacement values become relatively large. The linear theory is valid only to small displacements. From Fig 28.2 you can also notice that the beam does not maintain its original form. This means that the linear elasticity theory can not take into consideration all the necessary phenomena that are related to the problem, anymore. To be able to solve the problem we must use general elasticity theory. This is done in the following subsection.

Table 28.1: Displacements with different load values

q_0 [N/m ²]	$\max u $ [m]
-1.0e7	0.04862
-1.0e8	0.4862
-1.0e9	4.862

28.2 Solution with nonlinear model

Solvers: ElasticSolve

Tools: editor

Dimensions: 2D, Steady-state

Case definition

In the following the beam problem is solved with general elasticity theory. That is done by using the nonlinear elasticity solver of Elmer. In the case of homogenous elastic material the problem can be written into a following mathematical form

$$\left\{ \begin{array}{ll} -\text{div}[(I + \nabla u)\Sigma] = 0 & \text{in } \Omega \\ \Sigma = \lambda(\text{tr}E)I + 2\mu E & \text{in } \Omega \\ E = \frac{1}{2}(\nabla u^T + \nabla u + \nabla u^T \nabla u) & \text{in } \Omega \\ u = 0 & \text{on } \Gamma_4 \\ (I + \nabla u)\Sigma n = 0 & \text{on } \Gamma_1 \cup \Gamma_2 \\ (I + \nabla u)\Sigma n = -q & \text{on } \Gamma_3 \end{array} \right.$$

where u is the displacement vector, q is the given surface load, Σ is the second Piola-Kirchhoff stress tensor, λ and μ are the Lamé constants and E is the Green-St Venant strain tensor.

Solution procedure

The problem is solved here without the graphical user interface. Open the solver input file of the linear case, Beam.sif, and edit the following changes. Define nonlinear elasticity as the only equation

```
Equation 1
  Name = "Equation1"
  Nonlinear Elasticity = Logical True
End
```

Change the name correspondingly in the Solver block and add information about the procedure needed. Leave all other keywords on the solver block unchanged.

```
Solver 1
  Equation = "Nonlinear Elasticity"
  Procedure = "ElasticSolve" "ElasticSolver"
  ...
End
```

Finally, the force load may be changed in boundary condition 2 as

```
Force 2 = Variable Coordinate 1
  0 0
  1 -1.0000e+09
End
```

The problem may now be solved from the command line by typing `ElmerSolver`.

Results

Table 28.2: Maximum displacements with different load values calculated according to general elasticity theory and linear theory.

q_0 [N/m ²]	max $ u $ [m]	max $ u $ (linear) [m]
-1.0e ⁷	0.04890	0.04862
-1.0e ⁸	0.4532	0.4862
-1.0e ⁹	1.297	4.861

From table 28.2 you can see the difference between the results calculated according to nonlinear and linear theory. According to the linear theory the displacement increases linearly as the load increases. This can be seen clearly from the results. The last loading level (-1.0e9 N/m²) is fairly large and the beam would probably break under that load. So the value of displacement might be unrealistic in that case.

Tutorial 29

Incompressible flow passing a step

29.1 Solution with linear triangles

Directory: FlowStepIncompressible

Solvers: FlowSolve

Tools: ElmerFront

Dimensions: 2D, Steady-state

Case definition

A fluid, flowing past a step (see figure 29.1), has the density 1 kg/m and viscosity 0.01 kg/ms. The velocity of the fluid on the incoming boundary Γ_6 in the x-direction is 1 m/s and in the y-direction 0 m/s (see figure 29.1). On the outgoing boundary Γ_4 the velocity is 0 m/s in the y-direction and the pressure is 0 Pa. The problem is to solve the velocity field and the pressure change in Ω .

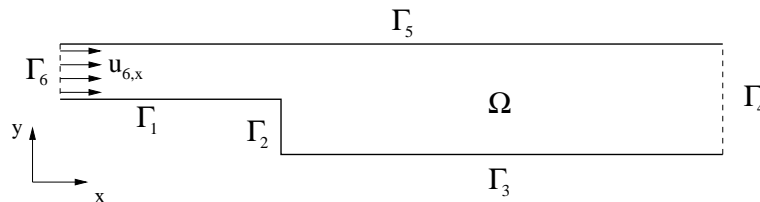


Figure 29.1: Step.

Mathematically the problem to be solved is

$$\begin{cases} -\nabla \cdot (2\mu\bar{\varepsilon}) + \rho\vec{u} \cdot \nabla\vec{u} + \nabla p = 0 & \text{in } \Omega \\ \nabla \cdot \vec{u} = 0 & \text{in } \Omega \end{cases} \quad (29.1)$$

with the boundary conditions

$$\begin{cases} \vec{u}_x = 1 & \text{on } \Gamma_6 \\ \vec{u}_x = 0 & \text{on } \Gamma_i, i = 1, 2, 3, 5 \\ \vec{u}_y = 0 & \text{on } \Gamma_i, i = 1, \dots, 6, \end{cases} \quad (29.2)$$

where μ is the viscosity, $\bar{\varepsilon}$ is the strain tensor, ρ is the density, \vec{u} is the velocity and p is the pressure. It is assumed that the density and viscosity are constants.

Solution procedure

- Start ElmerFront.

- Open the file that contains the geometry of the step from the File menu. Select also the working directory for the model.

```
File -> Open cad-file
File = StepFlow.egf
Model name = StepFlow
Model directory = step_tutorial
```

- Select the equations to be solved from the Problem menu. In this case we solve the Navier-Stokes equations.

```
Problem -> Equations
Navier-Stokes
```

- Define the material properties from the Model menu. Give the values for the density and the viscosity.

```
Model -> Materials
Density = 1
Viscosity = 0.01
```

- Define boundary conditions from the Model menu. Give the values of the velocities at each boundary Γ_i . Add the different boundary conditions to boundary condition sets and attach each constraint to a boundary or boundaries Γ_i that the constraint concerns (see figure 29.1).

```
Model -> Boundary conditions
on  $\Gamma_6$ : Velocity-X = 1 and Velocity-Y = 0
on  $\Gamma_i$ ,  $i=1,2,3,5$ : Velocity-X = 0 and Velocity-Y = 0
on  $\Gamma_4$ : Velocity-Y = 0
```

- Define mesh from the Mesh menu. First give name for the mesh and then define the element size. Create the mesh by pressing “Generate mesh” button.

```
Mesh -> Define mesh
Mesh name = Mesh1
Model Mesh H [m] = 0.2
Generate mesh
```

- Now to solve the problem select from the Run menu item Solver. This starts the solver.

```
Run -> Solver
```

- After the solution is done, view the results by selecting from the Run menu item Postprocessor.

```
Run -> Postprocessor
```

- To save the created model, select from the File menu item Save model file.

```
File -> Save model file
```

- To exit Elmer select from the File menu item Exit.

```
File -> Exit
```

Results

As a result the maximum pressure difference and maximum velocity is given (see table 29.1). One special result of interest is the point, on the x-axis, at which the direction of the flow changes. In this case its position is about 8.3 m.

Table 29.1: Pressure difference and velocity

Elements	$\max(\Delta p)$ [Pa]	$\max \vec{u} $ [m/s]
1426	1.039	1.489

29.2 Solution with 2nd order rectangles

Solvers: FlowSolve

Tools: ElmerGrid, editor

Dimensions: 2D, Steady-state

Case definition

In the following the flow past a step -problem is solved with eight-noded quadrilateral elements. The mesh is done with ElmerGrid which is a simple mesh generator that can be downloaded via the Elmer internet pages. Here a grd file is introduced. It contains the geometry data and parameters needed in defining the mesh of the structure. ElmerGrid transforms this file into Elmer mesh files (mesh.boundary, mesh.nodes, mesh.header and mesh.elements).

Solution procedure

The problem might be solved using ElmerFront by reading an external mesh into the program but here instructions for command line usage of Elmer are given.

- First generate mesh with ElmerGrid with the following command.

```
ElmerGrid 1 2 Step.grd
```

- Make the necessary changes to the .sif file. Changes are made to header section, boundary conditions and boundaries. The sif file is conveniently edited using a text editor. The sections should be edited into the following form

```
Header
  CHECK KEYWORDS Warn
  Mesh DB "." "Step"
End

Boundary Condition 1
  Name = "Constraint1"
  Target Boundaries(1) = 1

  Velocity 1 = 1
  Velocity 2 = 0
End

Boundary Condition 2
  Name = "Constraint2"
  Target Boundaries(1) = 3

  Velocity 1 = 0
  Velocity 2 = 0
End
```

```
Boundary Condition 3
  Name = "Constraint3"
  Target Boundaries(1) = 2

  Velocity 2 = 0
End
```

- To solve the problem run the solver by typing Solver.

Results

In Table 29.2 are the results of the problem solved with eight-noded quadrilateral (408) and three-noded triangular (303) elements.

Element type	Elements	$\max(\Delta p)$ [Pa]	$\max \vec{u} $ [m/s]
408	531	1.182	1.404
303	1416	1.039	1.489

Table 29.2: Pressure difference and maximum velocity with 2nd order rectangles and first order triangles.

When the problem is solved with eight-noded quadrilateral elements the point at which the flowing direction changes is about 8.5 m.

Tutorial 30

Transient flow and heat equations - the Rayleigh-Benard instability

Directory: RayleighBenard

Solvers: HeatSolve, FlowSolve

Tools: ElmerGrid, Editor

Dimensions: 2D, Transient

Case definition

This tutorial is about simulating the developing of the Rayleigh-Benard instability in a rectangular domain (Figure 30.1) of dimensions 0.01 m height and 0.06 m length. The simulation is performed with water and the needed material parameters of water are presented in Table 30.1. The temperature difference between the upper and lower boundary is set to 0.5 so that lower one has the temperature of 283.5 K and the upper one has the temperature of 283 K.

The density of water is inversely proportional to its temperature. Thus, heated water starts to flow upwards, and colder downwards due to gravity. In this case we assume that the Boussinesq approximation is valid for thermal incompressible fluid flow. In other words, the density of the term $\rho \vec{f}$ in the incompressible Navier-Stokes equation can be redefined by the Boussinesq approximation

$$\rho = \rho_0(1 - \beta(T - T_0))$$

where β is the heat expansion coefficient and the subscript 0 refers to a reference state.

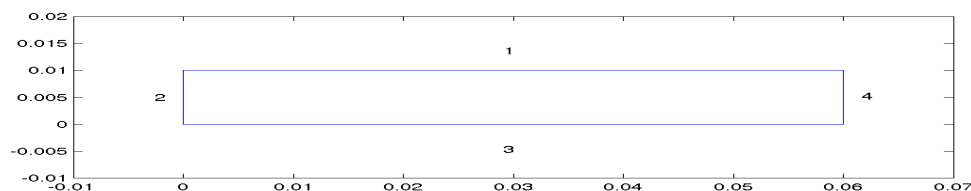


Figure 30.1: Domain.

Table 30.1: Material parameters.

parameter	value
density	1000 kg/m ³
viscosity	1040e-6 Ns/m ²
heat capacity	4190 J/(kg·K)
heat conductivity	0.6 W/(m·K)
heat expansion coefficient	1.8e-4 K ⁻¹
reference temperature	283 K

Solution procedure

The mesh has been constructed in advance and it consists of 2646 elements. The mesh files locate under the directory Mesh.

```
Header
  Mesh DB "." "Mesh"
  Include Path ""
  Results Directory ""
End
```

The simulation is carried out in 2-dimensional cartesian coordinates. Newmark time-stepping method is selected with 200 steps and with step size of two seconds. In addition, we have to set the value Transient for the keyword Simulation Type.

```
Simulation
  Coordinate System = "Cartesian 2D"
  Coordinate Mapping(3) = 1 2 3
  Timestepping Method = "Newmark"
  Newmark Beta = 1
  Timestep Intervals(1) = 200
  Timestep Sizes(1) = 2
  Output Intervals(1) = 1
  Simulation Type = "Transient"
  Steady State Max Iterations = 10
  Solver Input File = "RayleighBenard.sif"
  Output File = "RayleighBenard.result"
  Post File = "RayleighBenard.ep"
End
```

Only one body is needed during the simulation.

```
Body 1
  Equation = 1
  Material = 1
  Body force = 1
  Initial Condition = 1
End
```

We assume that the Boussinesq approximation is valid.

```
Body Force 1
  Boussinesq = True
End
```


In next section the needed initial conditions are defined.

```
Initial Condition 1
  Temperature = 283
  Velocity 1 = 1.0e-9
  Velocity 2 = 0
End
```

The equations of the model are coupled and that is why the convection is computed.

```
Equation 1
  Navier-Stokes = True
  Heat Equation = True
  Convection = "Computed"
End
```

We must also define the material parameters of water.

```
Material 1
  Density = 1000
  Viscosity = 1040e-6
  Heat Capacity = 4190
  Heat Conductivity = 0.6
  Heat Expansion Coefficient = 1.8e-4
  Reference Temperature = 283
End
```

The heat equation is solved first using direct solver.

```
Solver 1
  Equation = "Heat Equation"
  Stabilize = Logical True
  Nonlinear System Max Iterations = 1
  Nonlinear System Convergence Tolerance = 1.0e-6
  Nonlinear System Newton After Iterations = 3
  Nonlinear System Newton After Tolerance = 1.0e-3
  Nonlinear System Relaxation Factor = 1
  Linear System Solver = "Direct"
  Linear System Iterative Method = "BiCGStab"
  Linear System Convergence Tolerance = 1.0e-12
  Linear System Max Iterations = 100
  Steady State Convergence Tolerance = 1.0e-6
End
```

The second solver solves the incompressible Navier-Stokes equation. We have used iterative solver this time. Also all the convergence criterions are defined in the following section.

```
Solver 2
  Equation = "Navier-Stokes"
  Stabilize = Logical True
  Nonlinear System Max Iterations = 1
  Nonlinear System Convergence Tolerance = 1.0e-6
  Nonlinear System Newton After Iterations = 3
  Nonlinear System Newton After Tolerance = 1.0e-3
  Nonlinear System Relaxation Factor = 1
  Linear System Solver = "Iterative"
```

```
Linear System Iterative Method = "BiCGStab"  
Linear System Convergence Tolerance = 1.0e-9  
Linear System Max Iterations = 100  
Linear System Preconditioning = ILU1  
Steady State Convergence Tolerance = 1.0e-05  
End
```

Finally we have to specify the boundary conditions. The temperature difference between the top and bottom boundary (Boundary Conditions 1 and 3) is set to be 0.5 degrees. The velocities in x- and y-direction have to be zero in all the boundaries.

```
Boundary Condition 1  
  Target Boundaries = 1  
  Velocity 1 = 0  
  Velocity 2 = 0  
  Temperature = 283  
End
```

```
Boundary Condition 2  
  Target Boundaries = 2  
  Velocity 1 = 0  
  Velocity 2 = 0  
End
```

```
Boundary Condition 3  
  Target Boundaries = 3  
  Velocity 1 = 0  
  Velocity 2 = 0  
  Temperature = 283.5  
End
```

```
Boundary Condition 4  
  Target Boundaries = 4  
  Velocity 1 = 0  
  Velocity 2 = 0  
End
```

Results

Due to the number of the time-steps the simulation may easily take more than ten minutes. After the solver has finished, the results can be postprocessed with ElmerPost. When opening the result-file it is important to press the button **All** which selects all the calculated time steps. A video of the results can be viewed by selecting the option **Timestep Control** and pressing the button **Loop** under the **Edit** menu.

In Figures 30.2 and 30.3 the obtained temperature distribution and the velocity vectors are presented. We have also measured the maximum velocity of the flow (Table 30.2). However, more accurate presentation of the simulation results is presented in Elmer's example page in the internet address <http://www.csc.fi/elmer/examples/rayleighbenard/index.html>.

Table 30.2: Maximum velocity.

maximum velocity	3.98e-4 m/s
------------------	-------------

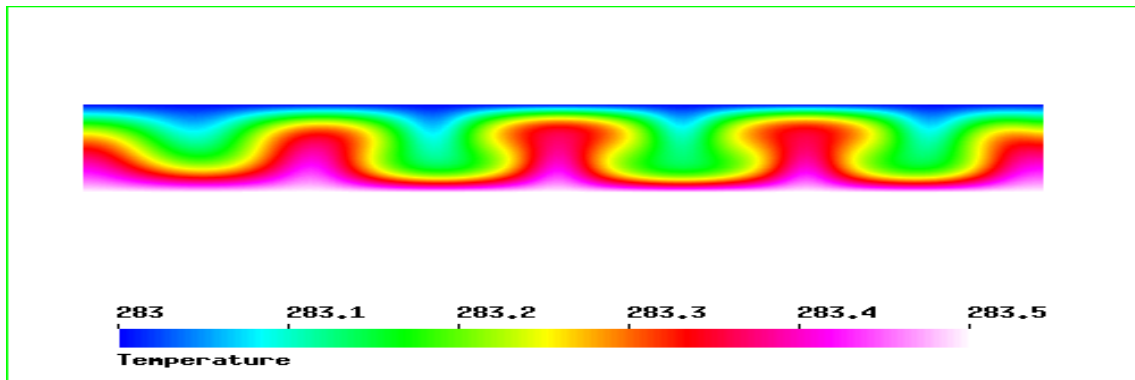


Figure 30.2: Temperature distribution at 260 s.

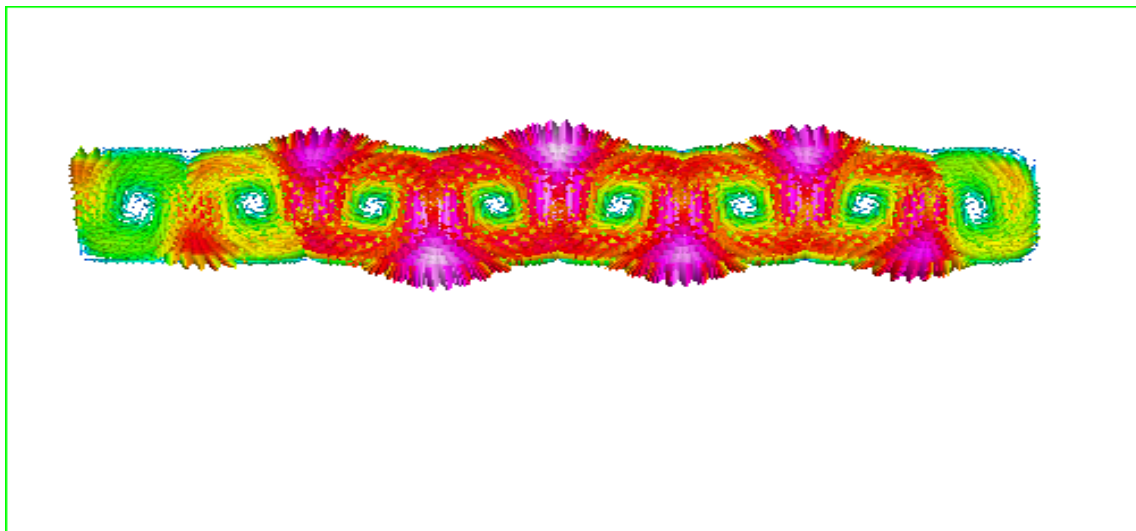


Figure 30.3: Velocity vectors at 260 s.

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